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A REVISED COMPUTER PROGRAM FOR AXIAL COMPRESSOR DESIGN.
VOLUME I. THEORY, DESCRIPTIONS, AND USER'S INSTRUCTIONS

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A revised computer program for the design of axial compressors is presented. It comprises three principal sections, two alternative means of determining blade geometry and an aerodynamic computation for the flow through the compressor. One method of determining blade geometry uses various analytic meanlines for the blade sections, and leads to the aerodynamic analysis of the flow through specified blading. The other

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method consists of creating arbitrary blade sections to follow the flow directions previously determined in an aerodynamic design calculation. The aerodynamic design section incorporates a loss calculation routine that may be used to estimate the design point performance of the compressor. One, two, or all three sections may be used in any one run of the program. This first volume of two describing the program details the theory of the method, describes the computer program, and gives all information necessary to use it.

# PREFACE

The work described in this Final Report was performed by the University of Dayton Research Institute, 300 College Park Avenue, Dayton, Ohio 45469, under Air Force Contract F33615-74-C-4030 during the period October 1973 to October 1974. It was a portion of Project 7065, Task 04 of the Aerospace Research Laboratories, and Project 3066 of the Aero Propulsion Laboratory. Technical Monitors for the Air Force were Dr. A. J. Wennerstrom, ARL/LF, and Mr. M. A. Stibich, AFAPL/TBC, both at Wright-Patterson Air Force Base, Ohio 45433. This report was initially identified as UDRI-TR-74-47 by the University of Dayton.

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# SECTION I

## INTRODUCTION AND REPORT ARRANGEMENT

This report, in two volumes, describes a computer program that has been developed for the design of axial compressors. It is an updated version of the program presented in References 1 and 2. The principal purpose of the program is to enable a single computer program to determine the geometry of the compressor blading, details of the flow within the compressor, and the design point performance of the machine. Some optional calculation routines will also enable effects of mixing of the flow to be investigated. The program consists fundamentally of three sections; two alternative means of determining blade geometry, and an aerodynamic computation for the flow through the compressor.

One method of determining blade geometry involves the use of analytic meanlines for the blade sections. This section of the program is essentially the computer program described in Reference 3, modified according to Reference 5. These modifications are the difference between this section of the new program and the same section of the original program of References 1 and 2, and extend the capabilities of the program section to include the optional generation of "splitter" blades instead of conventional "principal" blades, and also the optional output of data compatible with the NASTRAN stress analysis program. Output from the section optionally includes relative flow angles through the blades. These may be used as partial input data to the aerodynamic determination section of the program which will then yield the flow characteristics for the blading. An alternative input form to the aerodynamic section is the specification of the desired angular momentum distribution through the blade. Output from this calculation will include relative flow angles through the blade. These may be used as input to the third section of the program, the generation of blades having arbitrary section meanlines. This section is based upon the computer program described in Reference 4, modified according to Reference 5. The modifications described in Reference 5 extend the program section capabilities as for the analytic meanline section mentioned above, and further, the method of generating the meanline between the points prescribed by the aerodynamic section calculation has been improved. For both blading methods, flow deviation angles link the blade angles to relative flow angles.

The aerodynamic section was written anew for the original program of References 1 and 2, and has been modified somewhat in this revised version. The principal changes are the addition of an optional alternative form of momentum equation for the meridional velocity profiles and optional output of pressure force data in a format compatible with the

NASTRAN stress analysis program. The aerodynamic section is described in detail in this report, and sufficient information is given regarding the blading sections to enable them and the whole program to be used, but for finer details of the blading calculations, the references given above should be consulted.

The following three sections of this report give details of the capabilities and method of the program, the input data requirements, and the resulting output data. These sections should give the casual program user all information necessary to use the program. Section V details the theory of the methods used in the aerodynamic section. This will enable the aerodynamic flow model to be evaluated, and the various program limitations and capabilities to be more fully understood. The system of equations derived is necessarily solved numerically and iteratively, and the various numerical procedures involved are described in Section VI. Details of the FORTRAN programming, including implementation of the program on a particular computing system, are given in Section VII.

FORTRAN listings of the program and a sample run are shown in Volume II of the report.

## SECTION II

## SCOPE AND GENERAL METHOD OF PROGRAM

### 1. OVERALL PROGRAM

In one run of the program, any one section alone may be used or the aerodynamic section may be used in conjunction with either or both of the blading sections. When the blading sections are used alone, it is assumed that one blade only will be produced in each run. When they are used in conjunction with the aerodynamic section, any number of blades may be produced by either method, subject only to the limitations imposed by the number of computing stations the aerodynamic section can handle. When the analytic meanline blade section is used in conjunction with the aerodynamic section, the data from the blade calculations that are carried over to the aerodynamic analysis consists essentially of the blockage caused by the blade and the blade angles. Two angles are involved, the "section angle" defined by the intersection of the blade mean surface with a cylinder, and the "lean angle" defined by the intersection of the blade mean surface with a computing station. Additional data describing the blade performance are required before an aerodynamic analysis can be performed, namely, the deviation angles and a specification of the losses. This is combined with the data from the blading section in an interface routine and then the completed data are passed to the aerodynamic section. When the arbitrary meanline blade section is used following an aerodynamic design calculation, the data carried over to the blading calculation consist essentially of the blade section angles for the blade. The aerodynamic design calculation produces the relative flow angles, which, in a manner similar to that mentioned above, are combined with the flow deviation angles except that in this case the calculation was included in the blade program described in Reference 4.

The program is written in standard FORTRAN IV and should be immediately compatible with all medium to large computing systems. All development running was done on a CDC 6600 system incorporating CALCOMP software and on-line hardware at Wright-Patterson Air Force Base. The program uses overlay; each of the main sections uses close to 130K (octal) of central memory.

### 2. AERODYNAMIC SECTION

The compressor flow computed in the aerodynamic section is assumed to be axisymmetric and inviscid. The two alternative momentum equations include entropy gradients in both the cross streamwise and streamwise directions, and also the blade forces. The fluid properties are

all computed in a group of FUNCTION SUBPROGRAMS. As shown in this report, the properties are computed for a perfect gas. This may be changed by replacing these routines. The streamline curvature method of solution is employed to solve the system of equations. In this method, a number of computing stations are located at strategic points in the flow and preferably, but not necessarily, close to orthogonal to the local meridional streamline direction. Typically, several stations are situated upstream of the first blade row; one will be at the edge of each blade row and several will be downstream of the last blade row. Additionally, stations may be placed within the blade rows, and this is normally done when detailed blade evaluations are being performed for high speed blades. The stations may be defined quite freely; they need not be radial nor need they be straight lines. A computing mesh is formed by the intersection of these (fixed) stations with the streamlines, whose locations are iteratively determined. An initial estimate is made of the streamline locations. The flow within the compressor is computed on the basis of this estimate and the resulting flow distribution enables a new estimate to be made. This procedure is repeated until the estimated streamline pattern is correct (to within a given tolerance). Up to 30 computing stations and 21 streamlines may be used to describe the flow field.

Losses occurring for each blade row may be specified by means of the radial distribution of relative total pressure loss coefficient, isentropic efficiencies, or entropy increases. Methods of specifying the angular momentum fall into two classes, "design" and "analysis". For the design cases, the radial distribution of total pressure, total enthalpy, absolute angular momentum, or absolute whirl velocity may be given. For the analysis cases, the relative flow angle distribution is required and this may be specified directly, or a blade angle and deviation angle may be given. Relative flow angle or blade angle may be given in the streamline direction, or as would occur were the streamsurfaces all concentric cylinders. An off-design analysis option involving relative total pressure loss coefficients and relative flow angles is included. In this, a mimimum loss coefficient and a reference relative outlet flow angle are given and deviations from these values computed as a function of relative inlet angle and its reference value. In order to produce designs in which the blade row losses are consistent with a loss model (rather than being specified in the data and then updated manually between computer runs), a loss re-estimation procedure is incorporated which may be used to continuously update relative total pressure loss coefficient values as the computation proceeds. A second option using the same routine is to only use it to compute and display new loss coefficients after the main solution is computed. Although intended as a design calculation aid, this routine may be employed regardless of whether a design or analysis method of determining angular momentum is employed for the blade row.

Optionally, the blockage due to boundary layers on the annulus walls may be computed from a simple, attached turbulent boundary layer equation.

Provision is made in the procedure to incorporate two viscous flow effects, which are the turbulent mixing of the flow and the radial transfer of the blade wakes. Used in conjunction with the off-design analysis option mentioned above, this should enable the effect of these phenomena on multistage compressor performance to be investigated.

When the analytic meanline blade section is used to create blade geometry prior to execution of the aerodynamic section, an interface routine determines the flow deviation angles. Then the relative flow angles required by the aerodynamic section are the sum of the blade angles and the deviation angles. The deviation angles are calculated basically according to the method proposed in Reference 6, Equations 269 and 271 In this method, the deviation angles are composed of a reference deviation for a 10% thick uncambered section, modified for blade type and actual thickness, plus a component depending upon camber, inlet angle, solidity, and meanline form. Two extensions to the method have been made. First, the second component has been made a function also of point of maximum camber, and second, provisions have been made to add in an arbitrary additional component of deviation angle.

Optionally, output from the aerodynamic section is data describing the aerodynamic loading on the blade(s) in a format compatible with the input requirements of the NASTRAN stress analysis program. A convention has been established for locating the elements into which the blade is divided that has been followed in each of the three program sections where NASTRAN-compatible output is concerned. Reference 7 is a users manual for the NASTRAN program.

## 3. ANALYTIC MEANLINE BLADE SECTION

As mentioned previously, this section is basically the program described in Reference 3, modified according to Reference 5. The blade is defined by first producing a series of blade sections, one on each of a number of streamsurfaces (surfaces of revolution) through the blade, and then stacking them in a prescribed manner to produce the blade. "Manufacturing section" definitions in Cartesian coordinates may then be produced by interpolation at a series of planes perpendicular to the stacking axis.

Four alternative meanlines are available for the blade section. (Any one blade uses one meanline type throughout.) The first one consists of a fourth order polynomial in which the coefficients are determined by

satisfying the desired inlet and outlet angles and also specifying the ratios of second derivative at the inlet and outlet to the maximum value occurring on the section. This is particularly suited to blades having inlet Mach numbers in the range of approximately 0.8 to 1.7, where a relatively straight leading edge followed by increasing camber is desirable. The second meanline is composed of two exponential functions and allows additional degrees of freedom. Similar specifications are made at the inlet and outlet, and additionally, the location of the point where the two meanline equations meet and the blade angle at that point are specified. This enables "S" sections to be specified, in addition to sections having continuously positive camber or a straight leading edge region and a cambered aft portion. Thus, a blade may be produced that is potentially useful in the Mach number range of approximately 0.8 to over 2.0. The third meanline is simply a circular arc so that only the inlet and outlet angles may be defined. Blades based upon this meanline have been successfully used at subsonic and low transonic Mach numbers. The final meanline is composed of two circular arcs. Both the inlet and outlet angles and the location of the point where the two meanline segments meet and the blade angle at this point are required to be specified. This meanline has similar capabilities to the exponential meanline described above, and is currently in use in industry. In order to complete the blade section specification (with any of the above mentioned meanlines), a thickness definition is required. Two thickness specifications are used. The first consists of two third order polynomials, one applying from the leading edge to the point of maximum thickness, and the second to the remaining portion of the section. By varying the location of the point of maximum thickness, and the thickness at the section edges and at the point of maximum thickness, thickness distributions compatible with mechanical requirements and all but the lowest speed aerodynamic requirements may easily be produced. This thickness is applied to all meanlines except the (single) circular arc. However, this limitation can be overcome by using the multiple circular arc meanline with the extent of one arc set to some insignificant fraction of the total chord. For the case of the circular arc meanline only, the thickness distribution is set so that the two surfaces produced are also circular arcs, producing the so-called double circular arc section.

The stacking of the blade is achieved by locating each of the (two-dimensional) sections on the appropriate surface of revolution to form a three-dimensional blade definition. There are available three variations in the stacking method. The first is to stack the section (that is, pass the stacking axis, a radial line) through the two-dimensional section centroids, or points specified relative to the centroid. This will normally be used for rotor blades for reasons of stress. The other two methods are to stack the blade at either the leading or trailing edge.

The interpolation of sections in Cartesian coordinates, and therefore convenient for the manufacturing process, is achieved by fitting a spline curve through points on the blade surfaces and noting where it passes through the desired manufacturing planes.

Up to 80 points may be used to define each blade surface, and blade-sections on up to 21 streamsurfaces may be computed. The streamsurfaces may be defined at up to ten stations distributed upstream, within, and downstream of the blade. When the blade program-section is used in conjunction with the aerodynamic program-section, the streamsurfaces and stations are common to the two calculations, and hence, iteration between the two program-sections is required to achieve similarity between the streamsurfaces that are specified to the blading program-section and those generated in the ensuing aerodynamic analysis.

A further capability of the analytic meanline program section is the option to design "splitter" blades. Splitter blades are defined as blades having the same trailing edge location at the principal blades, a leading edge location somewhere between the leading and trailing edges of the principal blades, and the same meanline as that portion of the principal blades that they duplicate. (Typically, the chord of splitter blades will be about one half that of the principal blades.) Two thickness distributions may be used in conjunction with the meanline obtained in this way; the double-circular-arc (DCA) thickness distribution, and the "standard" double-polynomial thickness distribution described above for the principal blades.

Also, output data may be produced that can be used directly as input to the NASTRAN stress analysis program. Cards are created that describe the grid and elements that the blade has been divided into. These are geometric properties of the blade. Cards may also be produced that reflect the aerodynamic loading on the elements. As this program section is basically concerned with blade geometric properties, extra input is required to generate these loading figures. These may be derived from the output of a previous run of the aerodynamic section. Alternatively, the aerodynamic section itself may be directed to output the pressure-load data in NASTRAN-compatible form.

## 4. ARBITRARY MEANLINE BLADE SECTION

As mentioned previously, this program section is based upon the program described in Reference 4 and modified according to Reference 5. As for the Analytic Meanline case, the blade is defined by first producing a series of blade sections, one on each of a number of streamsurfaces,

and then stacking them to produce the blade. "Manufacturing section" definitions in Cartesian coordinates may then be produced by interpolation at a series of planes perpendicular to the stacking axis.

The unique feature of this blading method is the procedure used to produce the section meanlines. For each section, input to the calculation includes relative flow angles at a number of points along the meridional chordline. (These may either be input directly or have been produced by the aerodynamic section.) The incidence angle at the leading edge is specified and hence the blade section inlet angle is known. The trailing edge deviation angle is determined from an estimate of the cascade solidity, because the actual chord is, as yet, unknown. Then, a specification of the variation of deviation angle within the blade from the incidence angle at the leading edge to the trailing edge value yields the blade angle at each point where the relative flow angle was given. A spline-fit is made of the tangent of this angle distribution versus axially-projected chord length, and the resulting piece-wise cubic analytically integrated to yield a piece-wise quartic meanline. This is a departure from the method presented in the original program of References 1 and 2. In that case, a piece-wise cubic meanline was produced, but it was necessary to compute meanlines for a range of end-point conditions of the first cubic segment in order to find a satisfactory solution, and, in some cases, no feasible meanline was ever found. The new approach appears to produce an attractive meanline in all cases with no difficulty.

The length of the resulting meanline enables the cascade solidity estimate to be revised, and some iteration is normally required to get the trailing edge deviation angle consistent. This is performed automatically by the program.

The remainder of the procedure to produce the final blade definition is similar to that described for the Analytic Meanline Blade Section. The two-part cubic thickness distribution is applied to each meanline to define the section surfaces. Stacking and interpolation of "manufacturing sections" proceed as described above.

Splitter blades may be designed by this program section just as described for the Analytic Meanline Section, except that in this case only the thickness distribution comprising of two third-order polynomials is available.

Output for subsequent use with the NASTRAN stress analysis program may be produced just as for the Analytic Meanline Section.

## SECTION III

## INPUT DATA

### 1. DATA FORMAT

The input data consist of an initial indication of the number of entries that are to be made to each of three program sections, and then a data-set for each entry to each section. The data that are required for the interfacing of the output from the analytic meanline blade section to the aerodynamic section are included in the data-set for the analytic meanline section. Because input to the aerodynamic and arbitrary meanline sections may be generated by previous execution of the analytic meanline and aerodynamic sections, respectively, the input for these sections that are to be supplied directly by the user varies. This is indicated in the chart below by giving the FORTRAN variable name for the file from which any data are taken that are not always supplied directly.

LOG5 is the file from which input is taken that may be generated by the analytic meanline section. When the analytic meanline section has been directed to produce data for the aerodynamic section for a particular computing station, LOG5 becomes an internally generated scratchfile. Otherwise, LOG5 is attached to the standard input unit and the user supplies the data. LOG6 is similarly used for the arbitrary meanline section. If the aerodynamic section has been directed to produce appropriate data, these will be stored within the computer. Otherwise, the user supplies the data directly.

Three input formats are used; alphanumeric, real, and integer. Only one type of data occurs on any one card (with the exception noted below), and normal FORTRAN convention indicates which quantities are integer. The alphanumeric format is used for the four title cards. Up to 72 characters may be used, starting in column 1. Real numbers are punched in fields of 12 locations, starting in column 1 of the card, up to six numbers per card. Decimal points should be included to ensure correct interpretation of the data, and the numbers may be placed anywhere within the allocated fields. Integers are punched in fields of three locations, starting in column 1. No decimal point may be used, and the numbers must occupy the right-most locations of the allocated fields. The one exception referred to above is the last record defined for the aerodynamic section. It consists of three real numbers input in the usual manner in the first 36 columns, followed by two integers in the next three columns each. These data would normally be produced by a prior run of the program, and the integers are included merely to check that the cards have not become mis-ordered.

In the following chart, one line corresponds to one card (except where more than one line is required to complete the description of the card).

TITLE1

NANAL NAERO NARBIT

The following data-set is input to the analytic meanline section, and will occur NANAL times. The last record in this set is indicated with an asterisk.

TITLE2

NLINES NSTNS NZ NSPEC NPOINT NBLADE ISTAK

(Con) IPUNCH ISECN IFCORD IFPLOT IPRINT ISPLIT INAST

(Con) IRLE IRTE NSIGN

ZINNER ZOUTER SCALE STACKX PLTSZE

KPTS IFANGS

XSTA RSTA - Occurs KPTS times

R BLAFOR - Occurs NLINES times

Occurs NSTNS times

ZR B1 B2 PP QQ RLE

TC TE Z CORD DELX DELY

S BS - Only if ISECN = 1 or 3

Occurs NSPEC times

RLES TCS TES ZZS PERSPT - Occurs NSPEC times if ISPLIT ≥ 1

X1 X2 X3 XB - Occurs NLINES times for each station where IFANGS = 2 if ISPLIT ≥ 1

NRAD NDPTS NDATR NSWITC NLE NTE

XKSHPE SPEED

NOUT1 NOUT2 NOUT3 -Refers to leading edge station.

NR NTERP NMACH NLOSS NL1

(Con) NL2 NEVAL NCURVE NLITER NDEL

(Con) NOUT1 NOUT2 NOUT3 NBLAD

R XLOSS -Occurs NR times

Occurs for each station within blade or at trailing edge

RTE

DM DVFRAC -Occurs NDPTS times

Occurs NRAD times

* RDTE DELTAD AC-Occurs NDATR times

This group only occurs if NAERO = 1 or IPUNCH = 1

The following data-set is input to the aerodynamic section and will only occur if NAERO = 1. The last record in this set is indicated with a double asterisk.

TITLE3

CP GASR G EJ

NSTNS NSTRMS NMAX NFORCE NBL NCASE

(Con) NSPLIT NSET1 NSET2 NREAD NPUNCH NPLOT

(Con) NPAGE NTRANS NMIX NMANY NSTPLT NEQN

NWHICH - Occurs NMANY times

G EJ SCLFAC TOLNCE VISK SHAPE

XSCALE PSCALE RLOW PLOW XMMAX RCONST

CONTR CONMX

FLOW SPDFAC - Occurs NCASE times

NSPEC

XSTN RSTN - Occurs NSPEC times

Occurs NTNS times

NDATA NTERP NDIMEN NMACH

DATAC DATA1 DATA2 DATA3 -Occurs NDATA times

Inlet condition specification

(LOG5) NDATA NTERP NDIMEN NMACH NWORK

(Con) NLOSS NL1 NL2 NEVAL NCURVE NLITER

(Con) NDEL NOUT1 NOUT2 NOUT3 NBLADE

(LOG5) SPEED-If NDATA >0

(LOG5) DATAC DATA1 DATA2 DATA3 DATA4

(Con) DATA5

(LOG5) DATA6 DATA7 DATA8 DATA9

Occurs NDATA times

DELC DELTA - Occurs NDEL times

For stations 2 thru NSTNS

WBLOCK BBLOCK BDIST -Occurs NSTNS times

NDIFF

DIFF FDHUB FDMID FDTIP -Occurs NDIFF

Occurs NSET1 times

NM NRAD

TERAD

DM WFRAC -Occurs NM times

Occurs NRAD times

Occurs NSET2 times

DELF(1) DELF(2).... DELF (NSTRMS) - if NSPLIT = 1 or NREAD = 1

** R X XL II JJ - Occurs NSTRMS times for NSTNS stations if NREAD = 1

The following (and final) data-set is input to the arbitrary mean-line section, and will occur NARBIT times.

TITLE4

NLINES NSTNS NZ NSPEC ISEGPT NBLADE ISTAK

(Con) IPUNCH IFPLOT IPRINT ISPLIT INAST

ZINNER ZOUTER SCALE STACKX PLTSZE

IRLE IRTE NRADEV NINC NSIGN IFCA

XKSHPE SOLTOL

NPTS ] Occurs

RADEV ] NRADEV

SM DEVCRV -Occurs NPTS times ] times

RINC XINC DELDEV - Occurs NINC times

IFANGS ] Occurs

(LOG6) KPTS ] NSTNS

(LOG6) XSTA RSTA - Occurs KPTS times ] times

(LOG6) R AIRANG BLAFOR -Occurs NLINES times ]

ZR YA RLE TC TE ZZ ] Occurs NSPEC

DELX DELY ] times

RLES TCS TES ZZS PERSPT -Occurs NSPEC times if ISPLIT ≥ 1

X1 X2 X3 XB - Occurs NLINES times for each station where IFANGS = 2 if ISPLIT ≥ 1

## 2. DATA ITEM DEFINITIONS

The aerodynamic section may be used with any self-consistent unit system and, additionally, a "linear dimension scaling factor" (SCLFAC) is incorporated into the input so that some commonly used but inconsistent unit systems may be used. This is principally intended to allow the use of inches for physical dimensions and yet retain feet for velocities. The basic dimensions used in the data are length (L), time (T),

and force (F). Angles are expressed in degrees (A), and temperatures on an absolute temperature scale (D). Heat capacities (H) are also required. Some possible unit systems are given below, together with the corresponding value of SCLFAC.

| L | T | F | D | H | SCLFAC |
|---|---|---|---|---|---|
| Feet | Seconds | Pounds | Deg. Rankine | BTU | 1.0 |
| Inches | Seconds | Pounds | Deg. Rankine | BTU | 12.0 |
| Metres | Seconds | Kilograms | Deg. Kelvin | CHU | 1.0 |

Note that some data names are used in more than one section; care should be taken to consult the correct sub-division below for definitions.

a. Initial Directives

TITLE1 — This is a title card for the run.

NANAL — The number of blades to be generated by analytic meanline blade section.

NAERO — If NAERO = 0, the aerodynamic section will not be entered.

If NAERO = 1, the aerodynamic section will be entered.

NARBIT — The number of blades to be generated by the arbitrary meanline blade section.

b. Analytic Meanline Blade Section

For a more detailed discussion of the input to this section through item XB, see References 3 and 5. For this section, the dimensioned input is either in degrees (A) or in length (L).

TITLE2 — A title card for the analytic meanline section of the program.

NLINES — The number of streamsurfaces which are defined, and on which blade sections will be designed. Must satisfy $2 \leq \text{NLINES} \leq 21$.

NSTNS — The number of computing stations at which the streamsurface radii are specified. Must satisfy $3 \leq \text{NSTNS} \leq 10$.

NZ — The number of constant-z planes on which manufacturing (Cartesian) coordinates for the blade are required. Must satisfy $3 \leq \text{NZ} \leq 15$.

| | |
|---|---|
| NSPEC | The number of radially disposed points at which the parameters of the blade sections are specified. Must satisfy $1 \leq$ NSPEC $\leq 21$. |
| NPOINT | The number of points that will be generated to specify the pressure and suction surfaces of each blade section. Must satisfy $2 \leq$ NPOINT $\leq 80$. Generally, no less than 30 should be used. It will be advantageous to specify 80 points when precision plots of the sections are to be produced directly by the program. |
| NBLADE | The number of blades in the blade row. |
| ISTAK | If ISTAK = 0, the blade will be stacked at the leading edge.<br><br>If ISTAK = 1, the blade will be stacked at the trailing edge.<br><br>If ISTAK = 2, the blade will be stacked at, or offset from, the section centroid. |
| IPUNCH | If the data for subsequent aerodynamic analysis are computed (IFANGS = 1), they will also be punched out if IPUNCH = 1. If not required, set IPUNCH to zero. |
| ISECN | If ISECN = 0, the blade will be constructed using the polynomial camber line and the standard (i.e., double-cubic) thickness distribution.<br><br>If ISECN = 1, the exponential camber line and the standard thickness distribution will be used.<br><br>If ISECN = 2, the circular arc camber line and the double-circular-arc thickness distribution will be used.<br><br>If ISECN = 3, the multiple-circular-arc meanline and the standard thickness distribution will be used. |
| IFCORD | If IFCORD = 0, the meridional projections of the streamsurface blade section chords are specified.<br><br>If IFCORD = 1, the streamsurface blade section chords are specified. |

IFPLOT — Where CALCOMP software is incorporated into the computing system, IFPLOT specifies the creation of precision plots. (Further information regarding the requirements for this are given in the section entitled "Program Operation and Structure.")

If IFPLOT = 0, no plots will be produced.

If IFPLOT = 1, a plot of the streamsurface sections will be produced. All NLINES sections are shown superimposed. The origin for each section plot is offset from the centroid of the section by distances specified by DELX and DELY.

If IFPLOT = 2, a plot of the manufacturing sections will be produced. The origin is the blade stacking axis, and all NZ sections are shown superimposed.

If IFPLOT = 3, both of the plots described for IFPLOT = 1 and 2 will be produced.

If IFPLOT = 4, individual plots of each of the manufacturing sections will be produced. The axes are rotated clockwise by the section stagger angle for each plot, so that a large scale may be used.

IPRINT — The input data is always listed by the program. Details of the streamsurface and manufacturing sections are printed as prescribed by IPRINT.

If IPRINT = 0, details of streamsurface and manufacturing sections are printed.

If IPRINT = 1, details of streamsurface sections are printed.

If IPRINT = 2, details of manufacturing sections are printed.

If IPRINT = 3, details of neither streamsurface nor manufacturing sections are printed. (The interface data for use with the aerodynamic section of the program is still displayed.)

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| ISPLIT | If ISPLIT = 0, principal (that is, conventional) blades are to be designed.<br><br>If ISPLIT = 1, splitter blades having the standard thickness distribution are to be designed.<br><br>If ISPLIT = 2, splitter blades having the DCA thickness distribution are to be designed. |
| INAST | If INAST = 0, no NASTRAN-compatible data is generated.<br><br>If \|INAST\| = 3, NASTRAN-compatible data is generated using three-point averages.<br><br>If \|INAST\| = 4, NASTRAN-compatible data is generated using four-point averages.<br><br>If INAST is positive, both geometric and pressure-load data are output.<br><br>If INAST is negative, the pressure-load data is not output, and will have to be generated elsewhere, such as by the aerodynamic section.<br><br>See the Output Data description (Section IV, 1) for further details. |
| IRLE | The computing station number at the blade leading edge. |
| IRTE | The computing station number at the blade trailing edge. |
| NSIGN | Indicator used to sign blade pressure forces according to program sign conventions. For <u>compressor rotors</u>, if the machine rotates clockwise when viewed from the front, set NSIGN to 1; otherwise, set NSIGN to -1. For <u>compressor stators</u>, the two values given for NSIGN are reversed. |
| ZINNER, ZOUTER | The NZ manufacturing sections are equi-spaced between z equals ZINNER and ZOUTER. |
| SCALE | When precision plots are produced, SCALE is the scale factor employed. |

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| STACKX | This is the axial coordinate of the stacking axis for the blade, relative to the same origin as used for the station locations, XSTA. |
| PLTSZE | The size (inches) of the plotter to be used in the creation of precision plots. |
| KPTS | The number of points provided to specify the shape of a computing station. |
| | If KPTS = 1, the computing station is upright and linear. |
| | If KPTS = 2, the computing station is linear and either upright or inclined. |
| | If KPTS > 2, a spline curve is fit through the points provided to specify the shape of the station. |
| IFANGS | If IFANGS = 0, the calculations of the quantities required for aerodynamic analysis will be omitted at a particular computing station. |
| | If IFANGS = 1, these calculations will be performed at that station. |
| XSTA | An array of KPTS axial coordinates (relative to an arbitrary origin) which, together with RSTA, specify the shape of a particular computing station. |
| RSTA | An array of KPTS radii which, together with XSTA, specify the shape of a particular computing station. |
| R | The streamsurface radii at NLINES locations at each of the NSTNS stations. |
| BLAFOR | The difference in pressure between points on the blade pressure surface and suction surface. If zero is input, the output generated for the NASTRAN program will reflect no aerodynamic loading. The pressure difference should be in units of force per length squared, where length is the same unit as the dimensions of the blade under consideration |

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| ZR | The variation of properties of the streamsurface blade section is specified as a function of streamsurface number. The various quantities are then interpolated (or extrapolated) at each streamsurface. The streamsurfaces are numbered consecutively from the innermost outward, starting with 1.0. ZR must increase monotonically, there being NSPEC values in all. |
| B1 | The blade inlet angle. |
| B2 | The blade outlet angle. |
| PP | If ISECN = 0, PP is the ratio of the second derivative of the camber line at the leading edge to its maximum value. Must satisfy $-2.0 < PP < 1.0$.<br><br>If ISECN = 1, PP is the ratio of the second derivative of the camber line at the leading edge to its maximum value forward of the inflection point. Must satisfy $0.0 < PP \le 1.0$.<br><br>If ISECN = 2 or 3, PP is superfluous. |
| QQ | If ISECN = 0, QQ is the ratio of the second derivative of the camber line at the trailing edge to its maximum value. Must satisfy $0.0 \le QQ \le 1.0$.<br><br>If ISECN = 1, QQ is the ratio of the second derivative of the camber line at the trailing edge to its maximum value rearward of the inflection point. Must satisfy $0.0 < QQ \le 1.0$.<br><br>If ISECN = 2 or 3, QQ is superfluous. |
| RLE | The ratio of blade leading edge radius to chord. |
| TC | The ratio of blade maximum thickness to chord. |
| TE | The ratio of blade trailing edge half-thickness to chord.<br><br>If ISECN = 2, TE is superfluous. |
| Z | The location of the blade maximum thickness, as a fraction of camber line length from the leading edge.<br><br>If ISECN = 2, Z is superfluous. |

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| CORD | If IFCORD = 0, CORD is the meridional projection of the blade chord.<br><br>If IFCORD = 1, CORD is the blade chord. |
| DELX, DELY | The stacking axis passes through the streamsurface blade sections, offset from the centroids, leading, or trailing edge by DELX and DELY in the x and y directions respectively. |
| S, BS | If ISECN = 1 or 3, S and BS are used to specify the locations of the inflection point (as a fraction of the meridionally-projected chord length) and the change in camber angle from the leading edge to the inflection point. If the absolute value of the angle at the inflection point is larger than the absolute value of B1, BS should have the same sign as B1, otherwise, B1 and BS should be of opposite sign. |
| RLES | The ratio of splitter section leading edge radius to splitter chord. |
| TCS | The ratio of splitter section maximum thickness to splitter chord. |
| TES | The ratio of splitter section trailing-edge half-thickness to splitter chord. If ISPLIT = 2, this item is superfluous. |
| ZZS | The location of the splitter section maximum thickness, as a fraction of splitter camber line length. If ISPLIT = 2, this item is superfluous. |
| PERSPT | The ratio of desired splitter axial chord to main blade axial chord on the particular streamsurface. |
| X1, X2, X3 | These quantities are not read. They may be left blank. |
| XB | The blockage due to the principal blades that is to be added to that computed for the splitter blades. (Generally, cards containing these data in the correct format will have been produced by use of the IFANGS = 1 and IPUNCH = 1 options when computing the principal blades. Take care only to incorporate those cards that correspond to locations within the splitter blade envelope.) |

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| NRAD | The number of radii at which a distribution of the fraction of trailing edge deviation is input. Must satisfy $1 \le$ NRAD $\le 5$. |
| NDPTS | The number of points used to define each deviation curve. Must satisfy $1 \le$ NDPTS $\le 11$. |
| NDATR | The number of radii at which an additional deviation angle increment and the point of maximum camber are specified. Must satisfy $1 \le$ NDATR $\le 21$. |
| NSWITC | If NSWITC = 1, the deviation correlation parameter "m" for the NACA $(A_{10})$ meanline is used.<br><br>If NSWITC = 2, the deviation correlation parameter "m" for double-circular-arc blades is used. |
| NLE | Station number at leading edge. |
| NTE | Station number at trailing edge. |
| XKSHPE | The blade shape correction factor in the deviation rule. |
| SPEED | See definition for Aerodynamic Section. |
| NR | The number of radii where a "loss" is input. |
| NTERP, NMACH, NLOSS, NL1, NL2, NEVAL, NCURVE, NLITER, NDEL, NOUT1, NOUT2, NOUT3, NBLAD | See definition for Aerodynamic Section. |
| R | Radius at which loss is specified. |

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| XLOSS | Loss description. The form is prescribed by NLOSS; see aerodynamic section. |
| RTE | Radius at blade trailing edge where the following deviation fraction/chord curve applies.<br><br>If NRAD = 1, it has no significance. Must increase monotonically. |
| DM | The location on the meridional chord where the deviation fraction is given. Expressed as a fraction of the meridional chord from the leading edge. Must increase monotonically. |
| DVFRAC | Fraction of trailing-edge deviation that occurs at location DM. |
| RDTE | Radius at trailing edge where additional deviation and point of maximum camber are specified. |
| DELTAD | Additional deviation angle added to that determined by deviation rule. Input positive for conventionally positive deviation for both rotors and stators. |
| AC | Fraction of blade chord from leading edge where maximum camber occurs. |

c. Aerodynamic Section

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| TITLE3 | A title card for the aerodynamic section of the program. |
| CP | Specific heat at constant pressure. An input value of zero will be reset to 0.24. Units: H/F/D. |
| GASR | Gas constant. An input value of zero will be reset to 53.32. Units: L/SCLFAC/D. |
| G | Acceleration due to gravity. An input value of zero will be reset to 32.174. Units: L/SCLFAC/T/T. |
| EJ | Joules equivalent. An input value of zero will be reset to 778.16. Units: LF/SCLFAC/H. |
| NSTNS | Number of computing stations. Must satisfy $3 \le$ NSTNS $\le 30$. |

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| NSTRMS | Number of streamlines. Must satisfy $3 \le$ NSTRMS $\le 21$. An input value of zero will be reset to 11. |
| NMAX | Maximum number of passes through the iterative streamline determination procedure. An input value of zero will be reset to 40. |
| NFORCE | The first NFORCE passes are performed with arbitrary numbers inserted should any calculation produce impossible values. Thereafter, execution will cease, the calculation having "failed". An input value of zero will be reset to 10. |
| NBL | If NBL = 0, the annulus wall boundary layer blockage allowance will be held at the values prescribed by WBLOCK. |
| | If NBL = 1, blockage due to annulus wall boundary layers will be recalculated except at station 1. VISK and SHAPE are used in the calculation. |
| NCASE | Number of speed/flow combinations to be computed. Must satisfy $1 \le$ NCASE $\le 10$, except that an input value of zero will be reset to 1. |
| NSPLIT | If NSPLIT = 0, the flow distribution between the streamlines will be determined by the program so that roughly uniform increments of computing station will occur between the streamlines at station 1. |
| | If NSPLIT = 1, the flow distribution between the streamlines is read in (see DELF). |
| NSET1 | The blade loss coefficient re-evaluation option (specified by NEVAL) requires loss parameter/diffusion factor data. NSET1 sets of data are input, the set numbers being allocated according to the order in which they are input. Up to 4 sets may be input (see NDIFF). |
| NSET2 | When NLOSS = 4, the loss coefficients at the station are determined as a fraction of the value at the trailing edge. Then, NSET2 sets of curves are input to define this fraction at a function of radius and meridional chord. Up to 2 sets may be input (see NM). |

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| NREAD | If NREAD = 0, the initial streamline pattern estimate is generated by the program.<br><br>If NREAD = 1, the initial streamline pattern estimate and also the DELF values are read in. (See DELF, R, X, XL.) |
| NPUNCH | If NPUNCH = 0, no action is taken.<br><br>If NPUNCH = 1, the final streamline pattern computed for each running point calculation that did not fail will be punched. The DELF values are also punched. |
| NPLOT | If NPLOT = 0, no action is taken.<br><br>If NPLOT = 1, a CALCOMP plot of the static pressure distributions on hub, mid, and tip streamlines will be made for each point.<br><br>If NPLOT = 2, a CALCOMP plot of the final streamline pattern will be made for each point.<br><br>If NPLOT = 3, the plots made for NPLOT = 1 and 2 will both be made. (See XSCALE, PSCALE, RLOW, PLOW.) |
| NPAGE | The maximum number of lines printed per page. An input value of zero will be reset to 80. |
| NTRANS | If NTRANS = 0, no action is taken.<br><br>If NTRANS = 1, relative total pressure loss coefficients will be modified to account for radial transfer of wakes. See Section V. 11. |
| NMIX | If NMIX = 0, no action is taken.<br><br>If NMIX = 1, entropy, angular momentum, and total enthalpy distributions will be modified to account for turbulent mixing. See Section V. 12. |
| NMANY | The number of computing stations for which blade descriptive data is being generated by the analytic mean-line section. |

NSTPLT

If NSTPLT = 0, no action is taken.

If NSTPLT = 1, a line-printer plot of the changes made to the midstreamline '$r$' coordinate is made for each computing station. If more than 59 passes through the iterative procedure have been made, then the plots will show the changes for the last 59 passes. The graph should decay approximately exponentially towards zero, indicating that the streamline locations are stabilizing. Decaying oscillations are equally acceptable, but, growing oscillations show the need for heavier damping in the streamline relocation calculations, that is, a decrease in RCONST.

NEQN

This item controls the selection of the form of momentum equation that will be used to compute the meridional velocity distribution at each computing station. There are two basic forms, and for each case, one may select not to compute the terms relating to blade forces. (See also Section V. 1.)

If NEQN = 0, the momentum equation involves the differential form of the continuity equations and hence $(1-M_m^2)$ terms in the denominator. Streamwise gradients of entropy and angular momentum (blade forces) are computed within blades and at the blade edges (provided data that describe the blades are given). Elsewhere, streamwise entropy gradients only are included in a simpler form of the momentum equation, except that at the first and last computing station, all streamwise gradients are taken to be zero. This is generally the preferred option when computing stations are located within the blade rows.

If NEQN = 1, the momentum equation form is similar to that used when NEQN = 0, but angular momentum gradients (blade force terms) are nowhere computed. This generally is the preferred option when computing stations are located at the blade edges only.

If NEQN = 2, the momentum equation includes an explicit $dV_m/dm$ term instead of the $(1-M_m^2)$ denominator terms. All streamwise gradients (including blade force terms) are computed as for the case NEQN = 0. When computing stations are located within the blade rows, the results will generally be similar to those obtained with NEQN = 0, and solutions may be found that cannot be computed with NEQN = 0 due to high meridional Mach numbers.

If NEQN = 3, the momentum equation is similar to that used when NEQN = 1, but (as for the case NEQN = 1) no angular momentum gradients are computed. This may be used when computing stations are located only at the blade edges and high meridional Mach numbers preclude the use of NEQN = 1.

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| NWHICH | The numbers of each of the computing stations for which blade descriptive data is being generated by the analytic meanline section. |
| SCLFAC | Linear dimension scale factor. An input value of zero will be reset to 12.0. |
| TOLNCE | Basic tolerance in iterative calculation scheme. An input value of zero will be reset to 0.001. (See discussion of tolerance scheme in Section VI.) |
| VISK | Kinematic viscosity of gas (for annulus wall boundary layer calculations). An input value of zero will be reset to 0.00018. Units: LL/SCLFAC/SCLFAC/T. |
| SHAPE | Shape factor for annulus wall boundary layer calculations. An input value of zero will be reset to 0.7. |
| XSCALE | The scale used for the physical dimensions in the static pressure and streamline plots given as length unit per inch of plot. |
| PSCALE | The scale used to plot static pressure given as pressure unit per inch of plot. |
| RLOW | Minimum value of radius shown on streamline plots. Units: L. |
| PLOW | Minimum value of pressure shown on static pressure plots. Units: F/L/L. |
| XMMAX | The square of the Mach number that appears in the equation for the streamline relocation relaxation factor is limited to be not greater than XMMAX. Thus, at computing stations where the appropriate Mach number is high enough for the limit to be imposed, a decrease in XMMAX corresponds to an increase in damping. If a value of zero is input, it is reset to 0.6. |

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| RCONST | The constant in the equation for the streamline relocation relaxation factor. The value of 8.0 that the analysis yields is often too high for stability. If zero is input, it is reset to 6.0. |
| CONTR | The constant in the blade wake radial transfer calculations. |
| CONMX | The eddy viscosity for the turbulent mixing calculations. Units: $L^2/SCLFAC^2/T$. |
| FLOW | Compressor flow rate. Units: F/T. |
| SPDFAC | The speed of rotation of each computing station is SPDFAC times SPEED (I). The units for the product are revolutions/(60xT). |
| NSPEC | The number of points used to define a computing station. Must satisfy $2 \leq$ NSPEC $\leq 21$, and also the sum of NSPEC for all stations $\leq 150$. If 2 points are used, the station is a straight line. Otherwise, a spline-curve is fitted through the given points. |
| XSTN, RSTN | The axial and radial coordinates, respectively, of a point defining a computing station. The first point must be on the hub and the last point must be on the casing. Units: L. |
| NDATA | Number of points defining conditions or blade geometry at a computing station. Must satisfy $0 \leq$ NDATA $\leq 21$, and also the sum of NDATA for all stations $\leq 100$. |
| NTERP | If NTERP = 0, and NDATA $\geq 3$, interpolation of the data at the station is by spline-fit.<br><br>If NTERP = 1 (or NDATA $\leq 2$), interpolation is linear point-to-point. |
| NDIMEN | If NDIMEN = 0, the data are input as a function of radius.<br><br>If NDIMEN = 1, the data are input as a function of radius normalized with respect to tip radius.<br><br>If NDIMEN = 2, the data are input as a function of distance along the computing station from the hub. |

If NDIMEN = 3, the data are input as a function of distance along the computing station normalized with respect to the total computing station length.

NMACH If NMACH = 0, the subsonic solution to the continuity equation is sought.

If NMACH = 1, the supersonic solution to the continuity equation is sought. This should only be used at stations where the relative flow angle is specified, that is, NWORK = 5, 6, or 7.

DATAC The coordinate on the computing station, defined according to NDIMEN, where the following data items apply. Must increase monotonically. For dimensional cases, units are L.

DATA1 At Station 1 and if NWORK = 1, DATA1 is total pressure. Units: F/L/L.

If NWORK = 0 and the station is at a blade leading edge, by setting NDATA $\neq$ 0, the blade leading edge may be described. Then DATA1 is the blade angle measured in the cylindrical plane. Generally negative for a rotor, positive for a stator. (Define the blade lean angle (DATA3) also.) Units: A.

If NWORK = 2, DATA1 is total enthalpy. Units: H/F.

If NWORK = 3, DATA1 is angular momentum (radius times absolute whirl velocity). Units: LL/SCLFAC/T.

If NWORK = 4, DATA1 is absolute whirl velocity. Units: L/SCLFAC/T.

If NWORK = 5, DATA1 is blade angle measured in the streamsurface plane. Generally negative for a rotor, positive for a stator. If zero deviation is input, it becomes the relative flow angle. Units: A.

If NWORK = 6, DATA1 is the blade angle measured in the cylindrical plane. Generally negative for a rotor, positive for a stator. If zero deviation is input, it becomes, after correction for streamsurface orientation and station lean angle, the relative flow angle. Units: A.

If NWORK = 7, DATA1 is the reference relative outlet flow angle measured in the streamsurface plane. Generally negative for a rotor, positive for a stator. Units: A.

DATA2 — At Station 1, DATA2 is total temperature. Units: D.

If NLOSS = 1, DATA2 is the relative total pressure loss coefficient. The relative total pressure loss is measured from the station that is NL1 stations removed from the current station, NL1 being negative to indicate an upstream station. The relative dynamic head is determined NL2 stations removed from the current station, positive for a downstream station, negative for an upstream station.

If NLOSS = 2, DATA2 is the isentropic efficiency of compression relative to conditions NL1 stations removed, NL1 being negative to indicate an upstream station.

If NLOSS = 3, DATA2 is the entropy rise relative to the value NL1 stations removed, NL1 being negative to indicate an upstream station. Units: H/F/D.

If NLOSS = 4, DATA2 is not used, but a relative total pressure loss coefficient is determined from the trailing edge value and curve set number NCURVE of the NSET2 families of curves. NL1 and NL2 apply as for NLOSS = 1.

If NWORK = 7, DATA 2 is the reference (minimum) relative total pressure loss coefficient. NL1 and NL2 apply as for NLOSS = 1.

DATA3 — The blade lean angle measured from the projection of a radial line in the plane of the computing station, positive when the innermost portion of the blade precedes the outermost in the direction of rotor rotation. Units: A.

DATA4 — The fraction of the periphery that is blocked by the presence of the blades.

DATA5 — Cascade solidity. When a number of stations are used to describe the flow through a blade, values are only required at the trailing edge. (They are used in the loss coefficient re-estimation procedure, and to evaluate diffusion factors for the output.)

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| DATA6 | If NWORK = 5 or 6, DATA6 is the deviation angle measured in the streamsurface plane. Generally negative for a rotor, positive for a stator. Units: A.<br><br>If NWORK = 7, DATA6 is reference relative inlet angle, to which the minimum loss coefficient (DATA2) and the reference relative outlet angle (DATA7) correspond. Measured in the streamsurface plane and generally negative for a rotor, positive for a stator. Units: A. |
| DATA7 | If NWORK = 7, DATA7 is the rate of change of relative outlet angle with relative inlet angle. |
| DATA8 | If NWORK = 7, DATA8 is the relative inlet angle larger than the reference value at which the loss coefficient attains twice its reference value. Measured in the streamsurface plane. Units: A. |
| DATA9 | If NWORK = 7, DATA9 is the relative inlet angle smaller than the reference value at which the loss coefficient attains twice its reference value. Measured in the streamsurface plane. Units: A. |
| NWORK | If NWORK = 0, constant entropy, angular momentum, and total enthalpy exist along streamlines from the previous station. (If NMIX = 1, the distributions will be modified.)<br><br>If NWORK = 1, the total pressure distribution at the computing station is specified. Use for rotors only.<br><br>If NWORK = 2, the total enthalpy distribution at the computing station is specified. Use for rotors only.<br><br>If NWORK = 3, the absolute angular momentum distribution at the computing station is specified.<br><br>If NWORK = 4, the absolute whirl velocity distribution at the computing station is specified.<br><br>If NWORK = 5, the relative flow angle distribution at the station is specified by giving blade angles and deviation angles, both measured in the streamsurface plane.<br><br>If NWORK = 6, the relative flow angle distribution at the station is specified by giving the blade angles measured in the cylindrical plane, and the deviation angles measured in the streamsurface plane. |

If NWORK = 7, the relative flow angle and relative total pressure loss coefficient distributions are specified by means of an off-design analysis procedure. "Reference", "stalling", and "choking" relative inlet angles are specified. The minimum loss coefficient varies parabolically with the relative inlet angle so that it is twice the minimum value at the "stalling" or "choking" values. A maximum value of 0.5 is imposed. "Reference" relative outlet angles and the rate of change of outlet angle with inlet angle are specified, and the relative outlet angle varies linearly from the reference value with the relative inlet angle. NLOSS should be set to zero.

NLOSS — If NLOSS = 1, the relative total pressure loss coefficient distribution is specified.

If NLOSS = 2, the isentropic efficiency (for compression) distribution is specified.

If NLOSS = 3, the entropy rise distribution is specified.

If NLOSS = 4, the total pressure loss coefficient distribution is specified by use of curve-set NCURVE of the NSET2 families of curves giving the fraction of final (trailing edge) loss coefficient.

NL1 — The station from which the loss(in whatever form NLOSS specifies) is measured, is NL1 stations removed from the station being evaluated. NL1 is negative to indicate an upstream station.

NL2 — When a relative total pressure loss coefficient is used to specify losses, the relative dynamic head is taken NL2 stations removed from the station being evaluated. NL2 may be positive, zero, or negative; a positive value indicates a downstream station, a negative value indicates an upstream station.

NEVAL — If NEVAL = 0, no action is taken.

If NEVAL > 0, curve-set number NEVAL of the NSET1 families of curve giving diffusion loss parameter as a function of diffusion factor will be used to re-estimate the relative total pressure loss coefficient. NLOSS must be 1, and NL1 and NL2 must specify the leading edge of the blade. See also NDEL.

If NEVAL < 0, curve-set number | NEVAL | is used NEVAL > 0, except that the re-estimation is only made after the overall computation is completed (with the input losses). The resulting loss coefficients are displayed but not incorporated into the overall calculation. See also NDEL.

NCURVE — When NLOSS = 4, curve-set NCURVE of the NSET2 families of curves, specifying the fraction of trailing-edge loss coefficient as a function of meridional chord is used.

NLITER — When NEVAL > 0, up to NLITER re-estimations of the loss coefficient will be made at a given station during any one pass through the overall iterative procedure. Less than NLITER re-estimations will be made if the velocity profile is unchanged by re-estimating the loss coefficients. (See discussion of tolerance scheme in Section VI.)

NDEL — When NEVAL = 0, set NDEL to 0. When NEVAL $\neq$ 0, and NDEL > 0, a component of the re-estimated loss coefficient is a shock loss. The relative inlet Mach number is expanded (or compressed) through a Prandtl-Meyer expansion on the suction surface, and NDEL is the number of points at which the Prandtl-Meyer angle is given. If NDEL = 0, the shock loss is set to zero. Must satisfy $0 \le$ NDEL $\le 21$, and also the sum of NDEL for all stations $\le 100$.

NOUT1 — If NOUT1 = 0, no action is taken.

If NOUT1 = 1, cards will be punched that may be incorporated into the data deck for a subsequent run of the analytic meanline section. These are the (NLINES times NSTNS) cards specifying R.

NOUT2 — If NOUT2 = 0, no action is taken.

If NOUT2 = 1, output records are created that may be used as input to the arbitrary meanline section. These are records specifying KPTS, XSTA, RSTA, R, and AIRANG. If NARBIT = 0, the records will be issued as punch cards; If NARBIT = 1, the records will be stored within the computer on file LOG6.

NOUT3 — This data item controls the generation of NASTRAN-compatible pressure difference output for use in a subsequent blade stress analysis. For details of the triangular mesh that is used, see the Output Description in Section IV. 2. c.

If NOUT3 = 1, the station is at a blade leading edge.

If NOUT3 = 2, the station is at a blade trailing edge.

If NOUT3 = 3, the station is at the trailing edge of one blade, and at the leading edge of another.

If NOUT3 = 0, the station may be between blade rows, or within a blade row for which output is required, depending upon the use of NOUT3 $\neq 0$ elsewhere. See also description of NBLADE below.

NBLADE — This item is used in determining the pressure difference across the blade. The number of blades is | NBLADE |. If NBLADE is positive, "three-point averaging" is used to determine the pressure difference across each blade element. If NBLADE is negative, "four-point averaging" is used. (See the Output Description in Section IV. 2. c.) If NBLADE is input as zero, a value of +10 is used. At a leading edge, the value for the following station is used: elsewhere the value at a station applies to the interval upstream of the station. Thus by varying the sign of NBLADE, the averaging method used for the pressure forces may be varied for different axial segments of a blade row.

SPEED — This card is omitted if NDATA = 0. The speed of rotation of the blade. At a blade leading edge, it should be set to zero. The product SPDFAC times SPEED has units of revolutions/( T x 60).

DELC — The coordinate at which Prandtl-Meyer expansion angles are given. It defines the angle as a function of the dimensions of the leading edge station, in the manner specified by NDIMEN for the current, that is trailing edge station. Must increase monotonically. For dimensional cases, units are L.

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| DELTA | The Prandtl-Meyer expansion angles. A positive value implies expansion. If blade angles are given at the leading edge, the incidence angles are added to the value specified by DELTA. Units: A. (Blade angles are measured in the cylindrical plane.) |
| WBLOCK | A blockage factor that is incorporated into the continuity equation to account for annulus wall boundary layers. It is expressed as the fraction of total area at the computing station that is blocked. If NBL = 1, values (except at Station 1) are revised during computation, involving data items VISK and SHAPE. |
| BBLOCK, BDIST | A blockage factor is incorporated into the continuity equation that may be used to account for blade wakes or other effects. It varies linearly with distance along the computing station. BBLOCK is the value at mid-station (expressed as the fraction of the periphery blocked), and BDIST is the ratio of the value on the hub to the mid-value. |
| NDIFF | When NSET1 > 0, there are NDIFF points defining loss diffusion parameter as a function of diffusion factor. Must satisfy $1 \le$ NDIFF $\le 15$. |
| DIFF | The diffusion factor at which loss parameters are specified. Must increase monotonically. |
| FDHUB | Diffusion loss parameter at 10 per cent of the radial blade height. |
| FDMID | Diffusion loss parameter at 50 per cent of the radial blade height. |
| FDTIP | Diffusion loss parameter at 90 per cent of the radial blade height. |
| NM | When NSET2 > 0, there are NM points defining the fraction of trailing edge loss coefficient as a function of meridional chord. Must satisfy $1 \le$ NM $\le 11$. |
| NRAD | The number of radial locations where NM loss fraction/chord points are given. Must satisfy $1 \le$ NRAD $\le 5$. |

TERAD | The fraction of radial blade height at the trailing edge where the following loss fraction/chord curve applies. If NRAD = 1, it has no significance.

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| TERAD | The fraction of radial blade height at the trailing edge where the following loss fraction/chord curve applies. If NRAD = 1, it has no significance. |
| DM | The location on the meridional chord where the loss fraction is given. Expressed as a fraction of meridional chord from the leading edge. Must increase monotonically. |
| WFRAC | Fraction of trailing edge loss coefficient that occurs at location DM. |
| DELF | The fraction of the total flow that is to occur between the hub and each streamline. The hub and casing are included, so that the first value must be 0.0, and the last (NSTRM) value must be 1.0. |
| R | Estimated streamline radius. (These data are input from hub to tip for the first station, from hub to tip for the second station, and so on.) Units: L. |
| X | Estimated axial coordinate at intersection of streamline with computing station. Units: L. |
| XL | Estimated distance along computing station from hub to intersection of streamline with computing station. Units: L. |
| II, JJ | Station and streamline number. These are merely read in and printed out to give a check on the order of the cards, presumed to have been punched on previous run. |

d. Arbitrary Meanline Blade Section

For a more detailed discussion of the input to this section, see Reference 4. In this section, the dimensioned input is either in degrees (A), or length (L).

| | |
|---|---|
| TITLE4 | A title card for the arbitrary meanline section of the program. |
| NLINES | As for Analytic Meanline Section. |
| NSTNS | As for Analytic Meanline Section. |
| NZ | As for Analytic Meanline Section. |

| | |
|---|---|
| NSPEC | As for Analytic Meanline Section. |
| ISEGPT | The number of points to define each surface in each blade segment, including the end-points common to the adjacent segments. Must satisfy 2≤ ISEGPT and also (ISEGPT-1) times the number of segments ≤ 80. (The number of segments is IRTE - IRLE + 1.) |
| NBLADE | As for Analytic Meanline Section. |
| ISTAK | As for Analytic Meanline Section. |
| IPUNCH | As for Analytic Meanline Section. |
| IFPLOT | As for Analytic Meanline Section. |
| IPRINT | As for Analytic Meanline Section. |
| INAST | As for Analytic Meanline Section. |
| ZINNER | As for Analytic Meanline Section. |
| ZOUTER | As for Analytic Meanline Section. |
| SCALE | As for Analytic Meanline Section. |
| STACKX | As for Analytic Meanline Section. |
| PLTSZE | As for Analytic Meanline Section. |
| IRLE | Station number at blade leading edge. Note that the first station specified in the arbitrary meanline section is number 1. |
| IRTE | Station number at blade trailing edge. |
| NRADEV | The number of radii at which a distribution of the fraction of trailing edge deviation is input. Must satisfy 1 ≤ NRADEV ≤ 5. |
| NINC | The number of radii at which the incidence angle is specified. Must satisfy 1 ≤ NINC ≤ 5. |
| NSIGN | This is used to establish the sign convention. Normally, + 1 for stators and -1 for rotors. |

| | |
|---|---|
| IFCA | If IFCA = 1, the deviation correlation parameter "m" for the NACA ($A_{10}$) meanline is used. If IFCA = 2, the deviation correlation parameter "m" for double-circular-arc blades is used. |
| XKSHPE | The blade shape correction factor in the deviation rule. |
| SOLTOL | Solidity tolerance in iteration for deviation angle. |
| NPTS | The number of points defining the deviation curve. |
| RADEV | Radius at the blade trailing-edge where the following deviation fraction/chord curve applies. If NRADEV = 1, it has no significance. Must increase monotonically. |
| SM | The location on the meridional chord where the deviation fraction is given. Expressed as a fraction of the meridional chord from the leading edge. Must increase monotonically. |
| DEVCRV | Fraction of trailing-edge deviation that occurs at location SM. |
| RINC | Radius at which incidence angle and additional deviation angle is given. |
| XINC | Incidence angle at radius RINC. Input positive for conventionally-positive incidence for both rotors and stators. |
| DELDEV | Additional deviation angle added to that determined by deviation rule. Input positive for conventionally positive deviation for both rotors and stators. Applied on the streamsurface that passes through the leading edge station at radius RINC. |
| IFANGS | As for Analytic Meanline Section. |
| XPTS | As for Analytic Meanline Section. |
| XSTA | As for Analytic Meanline Section. |
| RSTA | As for Analytic Meanline Section. |
| R | Streamsurface radius. |

| | |
|---|---|
| AIRANG | Relative flow angle at radius R. Normally negative for rotors, positive for stators. |
| BLAFOR | As for Analytic Meanline Section. |
| ZR | The variations of properties of the streamsurface blade sections are specified as a function of streamsurface number. The streamsurfaces are numbered consecutively from the innermost outward. Must increase monotonically. |
| YA | The chord is multiplied by the factor (1-YA) when the solidity is determined, which is then used in the deviation angle calculations. |
| RLE | The ratio of blade leading-edge radius to the chord. |
| TC | The ratio of blade maximum thickness to chord. |
| TE | The ratio of blade trailing-edge half-thickness to chord. |
| ZZ | The point of blade maximum thickness, as a fraction of camber line length from the leading edge. |
| DELX, DELY | As for Analytic Meanline Section. |
| X1, X2 X3, XB | As for Analytic Meanline Section. |

## SECTION IV

## OUTPUT DATA

### 1. ANALYTIC MEANLINE SECTION

Printed output may be considered to consist of four sections; a printout of the input data, details of the blade sections on each streamsurface, a listing of quantities required for aerodynamic analysis, and details of the manufacturing sections determined on the constant-z planes. These are briefly described below. In the explanation which follows, parenthetical statements are understood to refer to the particular case of the double-circular-arc blade (ISECN = 2).

The input data printout includes all quantities read in, and is self-explanatory.

Details of the streamsurface blade sections are printed if IPRINT = 0 or 1. Listed first are the parameters defining the blade section. These are interpolated at the streamsurface from the tables read in. Then follow details of the blade section in "normalized" form. The blade section geometry is given for the section specified, except that the meridional projection of the chord is unity. For this section of the output, the coordinate origin is the blade leading edge. The following quantities are given: blade chord; stagger angle; camber angle; section area; location of the centroid of the section; second moments of area of the section about the centroid; orientation of the principal axes; and the principal second moments of area of the section about the centroid. Then are listed the coordinates of the camber line, the camber line angle, the section thickness, and the coordinates of the blade surfaces. NPOINT values are given.

A lineprinter plot of the normalized section follows. The scales for the plot are arranged so that the section just fills the page, so that the scales will generally differ from one plot to another. "Dimensional" details of the blade section are given next. The normalized data given previously is scaled to give a blade section as defined by IFCORD and CORD. For this section of the output, the coordinates are with respect to the blade stacking axis. The following quantities are given: blade chord; radius and location of center of leading (and trailing) edge(s); section area, the second moments of area of the section about the centroid and the principal second moments of area of the section about the centroid. The coordinates of NPOINT points on the blade surfaces are then listed, followed by the coordinates of 31 points distributed at (roughly) six degree intervals around the leading (and trailing) edges. Finally, the coordinates of the blade surfaces and points around the leading (and trailing) edge(s) is (are) shown in Cartesian form.

The quantities required for aerodynamic analysis are printed at all computing stations specified by the IFANGS parameter. The radius, blade section angle, blade lean angle, blade blockage, and relative angular location of the camber line are printed at each streamsurface intersection with the particular computing station. The blade section angle is measured in the cylindrical plane, and the blade lean angle is measured in the constant-axial-coordinate plane.

Details of the manufacturing sections are printed if IPRINT = 0 or 2. At each value of z specified by ZINNER, ZOUTER, and NZ, section properties and coordinates are given. The origin for the coordinates is the blade stacking axis. The following quantities are given: section area; the location of the centroid of the section; the second moments of area of the section about the centroid; the principal second moments of area of the section about the centroid; the orientation of the principal axes; and the section torsional constant. Then the coordinates of NPOINT points on the blade section surfaces are listed, followed by 31 points around the leading (and trailing) edge(s).

If NAERO = 1, the additional input and output required for, and generated by, the interface are also printed. (Apart from the input data printout, this is the only printed output when IPRINT = 3.)

If IPUNCH = 1 and NAERO = 0, the program punches the quantities required for aerodynamic analysis, together with identifying indices denoting station number and streamsurface number, on cards in the following format: 5 fields each of 12 locations for the quantities themselves, followed by 2 fields each of 3 locations for the indices.

If INAST $\neq 0$, cards are punched that may be used as input for the NASTRAN stress analysis program. For the purpose of stress analysis, the blade is divided into a number of triangular elements, each defined by three grid points. The intersections between computing stations and streamsurfaces are used as the grid points, and the grid points and element numbering scheme adopted through Program UD0300 is illustrated in Figure 1.

The NASTRAN input data format includes cards identified by the codes GRID, CTRIA2, PTRIA2, and PLOAD2. The data are fully described in Reference 7, but briefly, the GRID cards each define a grid point number and give the coordinates at the grid point, the CTRIA2 cards each define an element in terms of the three appropriate grid points (by number, and in a significant order), the PTRIA2 cards each give an average blade thickness for an element, and the PLOAD2 cards each give an average pressure loading for an element. If $|\text{INAST}| = 3$, the thicknesses and pressures are

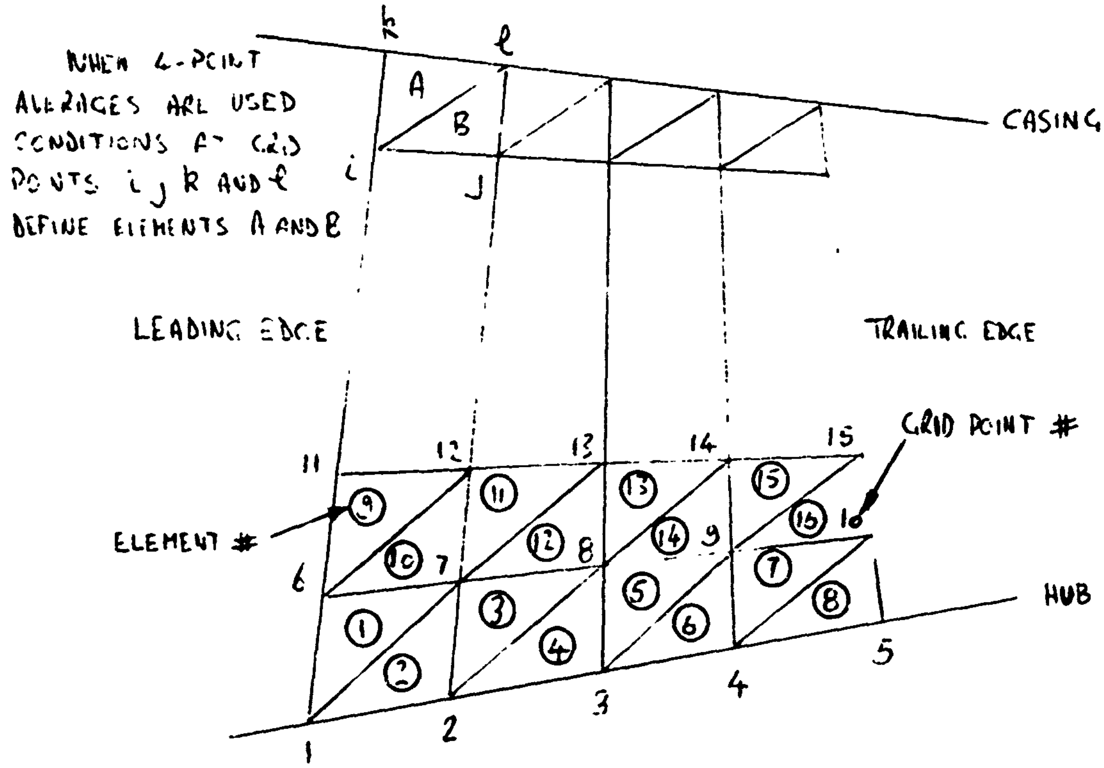


Figure 1. NASTRAN Grid Point and Element Numbering Scheme.

obtained by averaging the values at the three grid points that comprise the blade element. Because of the different weighting towards the "front" and "rear" of the elements, the averages computed do not always vary sensibly, especially near the blade edges, and therefore an alternative averaging procedure is also available. When |INAST| = 4, the averages are formed from values at the four grid points which are rather obviously related to the blade element. (This is indicated in Figure 1.) Note that as detailed in the input data description, if INAST is negative, the PLOAD2 cards are not punched.

Precision plots are produced if IFPLOT = 1, 2, 3, or 4 as described under the definition of IFPLOT given previously.

## 2. AERODYNAMIC SECTION

### a. Regular Printed Output

The input data are first printed out in its entirety, and the results for each running point follow. The output is generally self-explanatory and definitions are given here for some derived quantities. Tabular output is generally not started on a page unless it can be completed on the same page, according to the maximum number of lines permitted by the input variable NPAGE.

The results of each running point are given under a heading giving the running point number. Any diagnostics generated during the calculation will appear first under the heading. (Diagnostics are described in the following section.) Then, a station-by-station print out follows for each station through to the last station, or to the station where the calculation failed, if this occurred. One or more diagnostics will indicate the reason for the failure, in this event. Included in the meshpoint coordinate data is the distance along the computing station from the hub to the interception of the streamline with the station (L), and the station lean angle (GAMA). Where the radius of curvature of a streamline is shown as zero, the streamline has no curvature. The whirl angle is defined by

$$\tan \alpha = \frac{V_\theta}{V_m} \tag{1}$$

For stations within a blade, or at a blade trailing edge, a relative total pressure loss coefficient is shown. The loss of relative total pressure is computed from the station defined by the input variable NL1. If a loss coefficient was used in the input for the station (NLOSS = 1 or 4, or NWORK = 7), the input variable NL2 defines the station where the normalizing relative dynamic head is taken; otherwise, it is taken at the station defined by NL1. If the cascade solidity is given as anything but zero, it is used in the determination of diffusion factors. The following definition is used:

$$D = 1 - \frac{V_{2r}}{V_{1r}} + \frac{V_{\theta_{1r}} \quad V_{\theta_{2r}}}{2\sigma V_{1r}} \tag{2}$$

Inlet conditions (subscript 1) are taken from the station defined by the input variable NL1.

The last term in Equation 2 is multiplied by -1 if the blade speed is greater than zero, or the blade speed is zero and the preceding rotating blade row has negative rotation. This is necessary because relative whirl angles are (generally) negative for rotor blades and for stator blades that follow a rotor having "negative" wheel speed. Incidence and deviation angles are treated in the same way. so that positive and negative values have their conventional significance for all blades.

If annulus wall boundary layer computations were made (NBL = 1), details are shown for each station. Then, an overall result is given, including a statement of the number of passes that have been performed and whether the calculation is converged, unconverged, or failed. When the calculation is unconverged, the number of mesh points where the meridional velocity component has not remained constant to within the specified tolerance (TOLNCE) on the last two passes is shown as IVFAIL. Similarily, the number of streamtubes, defined by the hub and each streamline in turn, where the fraction of the flow is not within the same tolerance of the target value is shown as IFFAIL. If these numbers are small, say less than 10% of the maximum possible values, the results may generally be used. Otherwise, the computation should be rerun, either for a greater number of passes, or with modified relaxation factor constants. If the punch-streamline option (NPUNCH =1) was used, the streamline pattern generated in the first run may be input into the second run (with NREAD =1), restarting the calculation essentially where it was terminated and thus effecting a saving in computer time. The default option relaxation constants will generally be satisfactory but may need modification for some cases. If insufficient damping is specified by the constants, the streamlines generated will tend to oscillate and this may be detected by observing a relatively small radius of curvature for the mid-passage streamline that also changes sign from one station to the next. This may be corrected by rerunning the problem (from scratch) with a lower value input for RCONST, say, of 4.0 instead of 6.0. When the damping is excessive, the velocities will tend to remain constant while the streamlines will not adjust rapidly to the correct locations. This will be indicated by a small IVFAIL and a relatively large IFFAIL. For optimum program performance, RCONST should be increased, and the streamline pattern generated thus far could be used as a starting point. The second constant XMMAX (the maximum value of the square of Mach number used in the relaxation factor) is incorporated so that high subsonic or supersonic cases the damping does not decrease unacceptably. The default value of 0.6 may be too low for rapid program convergence in some such cases.

If the punching of blade pressure load data for the NASTRAN program is specified (by the input variable NOUT3), a self-explanatory printout is also made. The blade element numbering scheme is the same as that incorporated into both blading sections of the program, and illustrated in Figure :

If the loss coefficient re-estimation routine has been used for any bladerow(s) (NEVAL $\neq 0$), a printout summarizing the computations made will follow. A heading indicating whether the re-estimation was incorporated into the overall iterative procedure or whether it was merely made "after the event" is first printed. Then follows a self-explanatory tabulation of various quantities involved in the redetermination of the loss coefficient on each streamline.

b. Diagnostic Printed Output

The various diagnostic messages that may be produced by the aerodynamic section of the program are all shown. Where a computed value will occur, "x" is shown here.

JOB STOPPED - TOO MUCH INPUT DATA

The above message will occur if the sum of NSPEC or NDATA or NDEL for all stations is above the permitted limit. Execution ceases.

STATIC ENTHALPY BELOW LIMIT AT xxx. xxxxxExxx

The output routine (subroutine UD0311) calculates static enthalpy at each meshpoint when computing the various output parameters and this message will occur if a value below the limit (HMIN) occurs. The limiting value will be used, and the results printed become correspondingly arbitrary. HMIN is set in the Program UD03AR and should be maintained at some positive value well below any value that will be validly encountered in calculation.

PASSxxx STATIONxxx STREAMLINExxx PRANDTL-MEYER FUNCTION NOT CONVERGED - USE INLET MACH NO

The loss coefficient re-estimation procedure involves iteratively solving for the Mach number in the Prandtl-Meyer function. If the calculation does not converge in 20 attempts, the above message is printed, and as indicated, the Mach number following the expansion (or compression) is assumed to equal the inlet value. (The routine only prints output following the completion of all computations and printing of the station-by-station output data.)

PASSxxx STATIONxxx ITERATIONxxx STREAMLINExxx
MERIDIONAL VELOCITY UNCONVERGED VM = xx. xxxxxxExx
VM(OLD) = xx. xxxxxxExx

For "analysis" cases, that is at stations where relative flow angle is specified, the calculation of meridional velocity proceeds iteratively at each meshpoint from the mid-streamline to the case and then to the hub. The variable LPMAX (set to 10 in Subroutines UD0308 and UD0326) limits the maximum number of iterations that may be made at a streamline without the velocity being converged before the calculation proceeds to the next streamline. The above message will occur if all iterations are used without achieving convergence, and the pass number is greater than NFORCE. Convergence is here defined as occurring when the velocity repeats to within TOLNCE/5.0, applied nondimensionally. No other program action occurs.

PASSxxx STATIONxxx MOMENTUM AND/OR CONTINUITY UNCONVERGED W/W SPEC = xx.xxxxx VM/VM (OLD) HUB = xx.xxxxxMID=xx.xxxxx TIP = xx.xxxxx

If,following completion of all ITMAX iterations permitted for the flow rate or meridional velocity, the simultaneous solution of the momentum and continuity equations profile is unconverged, and the pass number is greater than NFORCE, the above message occurs. Here converged means that the flow rate equals the specified value, and the meridional velocity repeats, to within TOLNCE/5.0, applied nondimensionally. If loss coefficient re-estimation is specified (NEVAL > 0), an additional iteration is involved, and the tolerance is halved. No further program action occurs.

PASSxxx STATIONxxx VM PROFILE NOT CONVERGED WITH LOSS RECALC VM NEW/VM PREV HUB = xx.xxxxxx MID = xx.xxxxxx CASE = xx.xxxxxx

When loss re-estimation is specified (NEVAL> 0), up to NLITER solutions to the momentum and continuity equations are completed, each with a revised loss coefficient variation. If, when the pass number is greater than NFORCE, the velocity profile is not converged after the NLITER cycles of calculation have been performed, the above message is issued. For convergence, the meridional velocities must repeat to within TOLNCE/5.0, applied nondimensionally. No further program action occurs.

A further check on the convergence of this procedure is to compare the loss coefficients used on the final pass of calculation, and thus shown in the station-by-station results, with those shown in the output from the loss coefficient re-estimation routine, which are computed from the final velocities, etc.

PASSxxx STATIONxxx ITERATIONxxx STREAMTUBExxx STATIC ENTHALPY BELOW LIMIT IN MOMENTUM EQUATION AT xxx. xxxxxExxx

The static enthalpy is calculated (to find the static temperature) during computation of the "design" case momentum equation, that is, when whirl velocity is specified. If a value lower than HMIN (see discussion of second diagnostic message) is produced, the limiting value is inserted. If this occurs when IPASS > NFORCE, the above message is printed. If this occurs on the final iteration, the calculation is deemed to have failed, calculation ceases, and results are printed out through to this station.

PASSxxx STATIONxxx ITERATIONxxx STREAMTUBExxx LOOPxxx STATIC H IN MOMENTUM EQUN. BELOW LIMIT AT xxx. xxxxxExxx

This corresponds to the previous message, but for the "analysis" case. For failure, it must occur on the final iteration and loop.

PASSxxx STATIONxxx ITERATIONxxx STREAMTUBExxx MERIDIONAL MACH NUMBER ABOVE LIMIT AT xxx. xxxxxExx

When Subroutine UD0308 is selected (NEQN = 0 or 1), the meridional Mach number is calculated during computation of the design momentum equation, and a maximum value of 0.99 is permitted. If a higher value is calculated, the limiting value is inserted. If this occurs when IPASS > NFORCE, the above message is printed. If this occurs on the final iteration, the calculation is deemed to have failed, calculation ceases, and results are printed through to this station.

PASSxxx STATIONxxx ITERATIONxxx STREAMTUBExxx LOOPxxx MERIDIONAL MACH NUMBER ABOVE LIMIT AT xxx. xxxxxExxx

This corresponds to the previous message, but for the "analysis" case. For failure, it must occur at the final iteration and loop.

PASSxxx STATIONxxx ITERATIONxxx STREAMTUBExxx MOMENTUM EQUATION EXPONENT ABOVE LIMIT AT xxx. xxxxxExxx

An exponentiation is performed during the computation of the design case momentum equation, and the maximum value of the exponent is limited to 88.0. If this substitution is required when IPASS > NFORCE, the above message is printed. If it occurs on the final iteration, the calculation is deemed to have failed, calculation ceases, and results are printed through to this station.

PASSxxx STATIONxxx ITERATIONSxxx STREAMLINExxx
(MERIDIONAL VELOCITY) SQUARED BELOW LIMIT AT
xxx. xxxxxExxx.

If a meridional velocity, squared, of less than 1.0 is calculated during computation of the design-case momentum equation, this limit is imposed. If this occurs when IPASS>NFORCE, the above message is printed. If this occurs on the final iteration, the calculation is deemed to have failed, calculation ceases, and results are printed out through to this station.

PASSxxx STATIONxxx ITERATIONxxx STREAMLINExxx LOOPxxx
(MERIDIONAL VELOCITY) SQUARED BELOW LIMIT AT
xxx. xxxxxExxx.

This corresponds to the previous message, but for the "analysis" case. For failure, it must occur on the last iteration and loop.

PASSxxx STATIONxxx ITERATIONxxx STREAMTUBExxx
STATIC ENTHALPY BELOW LIMIT IN CONTINUITY EQUATION
AT xxx. xxxxxExxx.

The static enthalpy is calculated during computation of the continuity equation. If a value lower than HMIN (see discussion of second diagnostic message) is produced, the limiting value is imposed. If this occurs when IPASS>NFORCE, the above message is printed. If this occurs on the final iteration, the calculation is deemed to have failed, calculation ceases, and results are printed out through to this station.

PASSxxx STATIONxxx ITERATIONxxx STREAMLINExxx
MERIDIONAL VELOCITY BELOW LIMIT IN CONTINUITY AT
xxx. xxxxxExxx.

If a meridional velocity of less than 1.0 is calculated when the velocity profile is incremented by the amount estimated to be required to satisfy continuity, this limit is imposed. If this occurs when IPASS > NFORCE, the above message is printed. If this occurs on the final iteration, the calculation is deemed to have failed, calculation ceases, and results are printed through to this station.

PASSxxx STATIONxxx ITERATIONxxx OTHER CONTINUITY
EQUATION BRANCH REQUIRED

If when IPASS>NFORCE, a velocity profile is produced that corresponds to a subsonic solution to the continuity equation when a supersonic solution is required, or vice versa, the above message is printed. If this occurs on the final iteration, failure is deemed to have occurred, calculation ceases, and results are printed out through to this station.

PASSxxx STATIONxxx ITERATIONxxx STREAMLINExxx
MERIDIONAL VELOCITY GREATER THAN TWICE MID VALUE

During integration of the "design" momentum equations, no meridional velocity is permitted to be greater than twice the value on the mid-streamline. If this occurs when IPASS>NFORCE, the above message is printed. If this occurs on the final iteration, the calculation is deemed to have failed, calculation ceases, and results are printed through to this station. In the event that this limit interferes with a valid velocity profile, the constants that appear on cards $08$. 272, $08$. 279, $26$. 229, and $26$. 236 may be modified accordingly. Note that as the calculation is at this point working with the square of the meridional velocity, the constant for a limit of 2. 0 times the mid-streamline value, for instance, appears as 4. 0.

PASSxxx STATIONxxx ITERATIONxxx STREAMLINExxx
LOOPxxx MERIDIONAL VELOCITY ABOVE LIMIT xxxxxExx
LIMIT = xxxxxExx.

During integration of the "analysis" momentum equations, no meridional velocity is permitted to be greater than three times the value on the mid-streamline. If this occurs when IPASS>NFORCE, the above message is printed. If this occurs on the final loop of the final iteration, the calculation is deemed to have failed, calculation ceases, and results are printed through to this station. In the event that the limit interferes with a valid velocity profile, the constants that appear on cards $08$. 398, $08$. 409, $26$. 323, $26$. 334, and $26$. 329 may be modified accordingly. In each case <u>except</u> that of the last card noted, the program is working with meridional velocity squared, so that a limit of, for instance, 3. 0 times the mid-streamline value appears as 9. 0.

PASSxxx STATIONxxx STREAMLINExxx LIMITING MERIDIONAL
VELOCITY SQUARED = xxxxxExx.

In the Subroutine UD0308 (NEQN= 0 or 1), a maximum permissable meridional velocity (equal to the speed of sound) is established for each streamline at the beginning of each pass. The calculation yields the square of the velocity, and if a value of less than 1. 0 is obtained, a value of 6250000. 0 is superimposed (which corresponds to a meridional velocity of 2500. 0). If this occurs when IPASS>NFORCE, the above message is printed, and the calculation is deemed to have failed. Calculation ceases after the station computations are made, and results are printed through to this station.

PASSxxx STATIONxxx ITERATIONxxx STREAMLINExxx MERIDIONAL VELOCITY ABOVE SOUND SPEED VM = xxxx.xx A = xxxx.xx.

In Subroutine UD0308 (NEQN = 0 or 1), no meridional velocity is permitted to be larger than the speed of sound. The above message will occur if this limit is violated during integration of the "design" momentum when IPASS > NFORCE. If the limit is violated at any point when IPASS> NFORCE and on the last permitted iteration (last permitted loop also in the case of the "analysis" momentum equation), the calculation is deemed to have failed. Calculation ceases, and the results are printed through to this station.

MIXING CALCULATION FAILURE NO. n

The above message occurs when flow mixing calculations are specified, and the computation fails. The overall calculation is halted, and results are printed through to the station that is the upstream boundary for the mixing interval in which the failure occurred. The integer n takes on different values to indicate the specific problems as follows.

n = 1 In solving for the static pressure distribution at the upstream boundary of each mixing step, the average static enthalpy is determined in each streamtube (defined by an adjacent pair of streamlines). This failure indicates that a value less than HMIN was determined.

n = 2 Calculation of the static pressure distribution at the upstream boundary of the mixing step is iterative. This failure indicates that the procedure was not converged after 10 iterations.

n = 3 The static enthalpy on each streamline at the mixing step upstream boundary is determined from the static pressure and entropy there. This failure indicates that a value less than HMIN was determined.

n = 4 The axial velocity distribution at the mixing step upstream boundary is determined from the total enthalpy, static enthalpy, and tangential velocity distributions. This failure indicates that a value less than VMIN was determined.

n = 5 In solving for the static pressure distribution at the downstream boundary of each mixing step, the average static enthalpy is determined in each streamtube (defined by an adjacent pair of streamlines). This failure indicates that a value less than HMIN was determined.

n = 6 Calculation of the static pressure distribution at the downstream boundary of the mixing step is iterative. This failure indicates that the procedure was not converged after 10 iterations.

n = 7 The static enthalpy distribution at the mixing step downstream boundary is found from the total enthalpy, axial velocity, and tangential velocity distributions. This failure indicates that a value less than HMIN was determined.

n = 8 In order to satisfy continuity, the static pressure level at the mixing step downstream boundary is iteratively determined. This failure indicates that after 15 attempts, the procedure was unconverged.

c. Punched Card Output

Four output options may result in punched cards being produced by the aerodynamic section of the program. Use of the input item NOUT3 gives "PLOAD2 - Cards" punched in a format compatible with the NASTRAN stress analysis program. For the purposes of stress analysis, the blade is taken to be composed of a number of triangular elements. Two such elements are formed by the quadrilateral defined by two adjacent streamlines and two adjacent computing stations. The way that each quadrilateral is divided into two triangles, and the element numbering scheme that is used, are illustrated in Figure 1. The pressure difference for each element is given by an average of either three or four values at surrounding meshpoints. The pressure difference at each meshpoint is computed from the equation

$$\Delta p = \frac{2\pi r \rho}{N}\left\{\sin\beta\cos\beta\, g J t \frac{dS}{dm} + \frac{V_m}{r}\frac{d}{dm}(rV_\theta)\right\} \tag{3}$$

and as follows. At the blade leading edge (NOUT3 = 1 or 3), a forward difference is used to determine the meridional gradients. At the blade trailing edge (NOUT3 = 2 or 3), the pressure difference is taken to be zero. At stations with the bladerow (NOUT3 = 0, following a leading edge), mean central differences are used to determine the meridional gradients. When the input item NBLADE is positive (or zero) for a particular blade axial segment, then three-point averaging is used. For instance, for element number 1 in Figure 1, pressure differences at grid points 1, 6, and 7 would be used. If NBLADE is negative, four-point averaging is used. For instance for element number 1 in Figure 1, pressure differences at grid points 1, 2, 6, and 7 would be used. The same average would also apply to element number 2.

If NPUNCH = 1, cards will be punched that may be used as input when the NREAD = 1 option is used. These comprise one or more cards defining the DELF values, and then (NSTRMS times NSTNS) cards giving the streamline coordinates. These may be incorporated directly into a subsequent input data deck. Note that one set will be punched for every running point that does not fail.

For any station for which NOUT1 = 1, NSTRMS cards giving the streamline radius (R) will be punched that then may be used in a subsequent input data deck for the analytic meanline blade section. A set will be punched for each running point that does not fail, immediately following the NPUNCH = 1 cards for the running point, if this option is also specified.

The input item NOUT2 controls the final possible card output. If NARBIT ≠ 0, this output is stored within the computer system on file LOG6; otherwise, it will be placed on the punch file. For any station where NOUT2 ≠ 0, data are output that give KPTS, XSTA, and RSTA (that define the station geometry), and R and AIRANG (that define the streamsurface radii and relative air angles) for possible subsequent use in the arbitary meanline section of the progrm. (Note that BLAFOR is left blank (zero); the first punch option described above will give PLOAD2-cards for subsequent use with the NASTRAN program if desired.)

d. Precision Plot Output

Plots of the static pressure distribution and final streamline pattern may be made for each running point according to the input variable NPLOT. The program assumes the availability of CALCOMP software to generate instructions for a CALCOMP plotter. Generally, an 11-inch plotter would be used, and the input variables XSCALE, PSCALE, RLOW, and PLOW should be set accordingly.

## 3. ARBITRARY MEANLINE SECTION

Printed output from this section of the program may be considered to consist of four sections; a printout of the input data; details of the blade sections on each stream surface; a listing of quantities required for aerodynamic analysis; and details of the manufacturing sections determined on the constant-z planes. These are briefly described below.

The input data printout includes all quantities read in, and is self-explanatory.

Details of the streamsurface blade sections are printed if IPRINT = 0 or 1. The iterative determination of a consistent deviation angle and cascade solidity is recorded by the successive values of deviation and solidity that are produced. Next the camberline that meets the angle/meridional coordinate specification thus created is detailed, camberline coordinates, first and second derivatives, and radius of curvature being shown. Also shown are the camberline angles and meridional coordinates for which the camberline was computed. Then follow details of the blade section in "normalized" form. The blade section geometry is given for the section specified, except that the meridional projection of the chord is unity. For this section of the output, the coordinate origin is the blade leading edge. The following quantities are given: blade chord; stagger angle; camber angle; section area; location of centroid of the section; second moments of area of the section about the centroid; orientation of the principal axes; and the principal second moments of area of the section about the centroid. Then are listed the coordinates of the camber line, the camber line angle, the section thickness, and the coordinates of the blade surfaces. A line-printer plot of the normalized section follows. The scales for the plot are arranged so that the section just fills the page, so that the scales will generally differ from one plot to another. "Dimensional" details of the blade section are given next. The normalized data given previously are scaled to give the proper blade section. For this section of the output, the coordinates are with respect to the blade stacking axis. The following quantities are given: blade chord; radius and location of center of the leading edge; section area; the second moments of area of the section about the centroid. The coordinates of points on the blade surfaces are then listed, followed by the coordinates of 31 points distributed at six degree intervals around the leading edge. Finally, the coordinates of the blade surfaces and points around the leading edge are shown in Cartesian form.

The quantities required for aerodynamic analysis are printed at all computing stations specified by the IFANGS parameter. The radius, blade section angle, blade lean angle, blade blockage, and relative angular location at the camber line are printed at each streamsurface intersection with the particular computing section.

Details of the manufacturing sections are printed if IPRINT = 0 or 2.

At each value of z specified by ZINNER, ZOUTER, and NZ, section properties and coordinates are given. The origin for the coordinates is the blade stacking axis. The following quantities are given: section area; the location of the centroid of the section; the second moments of area of the section about the centroid; the principal second moments of area of the section about the centroid; the orientation of the principal axes; and the section torsional constant. Then the coordinates of points on the blade section surfaces are listed, followed by 31 points around the leading edge.

Precision plots are produced if IFPLOT = 1, 2, 3, or 4 as described under the definition of IFPLOT given previously.

If IPUNCH = 1, the program punches the quantities required for aerodynamic analysis, together with identifying indices denoting station number and streamsurface number, on cards in the following format: 5 fields each of 12 locations for the quantities themselves, followed by 2 fields each of 3 locations for the indices.

If INAST $\neq$ 0, punched cards are output for use with the NASTRAN stress analysis program. See the description given earlier for the corresponding output from the analytic meanline section of the program for details.

SECTION V

THEORY OF AERODYNAMIC ANALYSIS

## 1. MOMENTUM EQUATION

The meridional velocity distribution along each computing station is found by integrating a momentum equation. (The continuity equation supplies the necessary constant of integration.) Two different, basic momentum equations are included in the program, and for each of them there are forms that correspond to flow in the bladed and nonbladed regions of the compressors. Thus there are four momentum equations in all, and selection of the one (or two) appropriate to an application is controlled by the input variable NEQN. Each of the four momentum equations may be written (and solved) in two ways, depending upon whether the calculation is of the "analysis" or "design" type. In the analysis case, the relative flow angle ($\beta$) is fixed, whereas in the design case, it is the absolute angular momentum ($rV_\theta$) that is fixed. One application of the program will usually involve two forms of momentum equation (for the bladed and nonbladed regions of the compressor), and some analysis-type and some design-type calculations, so-called, regardless of whether the computation is actually a compressor design or analysis exercise.

The starting point for the development of the various equations is presented by Wennerstrom in Reference 8, Equation 14, and reproduced here. (Our sign convention for the station lean angle ($\gamma$) is the reverse of his.)

$$V_m \frac{dV_m}{d\ell} = \sin(\phi+\gamma) V_m \frac{dV_m}{dm} + \cos(\phi+\gamma)\frac{V_m^2}{r_c} - \frac{W_\theta}{r}\frac{d}{d\ell}(rW_\theta)$$

$$- 2\omega W_\theta \cos\gamma + gJ\frac{dI}{d\ell} - gJ\frac{dS}{d\ell} - \sin(\phi+\gamma)F_m - \cos(\phi+\gamma)F_n \qquad (4)$$

$F_m$ is a force acting in the meridional plane and in the (projected) streamline direction. $F_n$ is a force also in the meridional plane and normal to $F_m$. Equation 4 is our first basic momentum equation. In Reference 8, Wennerstrom defines the forces $F_m$ and $F_n$ in the context of the bladed regions of a turbomachine. His Equations 3 and 19 define $F_m$ thus:

$$F_m = \frac{W_\theta}{r}\frac{d(rV_\theta)}{dm} - gJt\frac{dS}{dm} \tag{5}$$

$F_n$ is defined by Wennerstrom's Equations 25 and 27 as follows:

$$F_n = \left(\sec\beta \sec\varepsilon \frac{V_m}{r}\frac{d(rV_\theta)}{dm} + \sin\beta \sec\varepsilon \, gJt\frac{dS}{dm}\right) \times$$

$$\left(\cos\beta \sin\varepsilon \sec(\phi+\gamma) + \sin\beta \cos\varepsilon \tan(\phi+\gamma)\right) \tag{6}$$

The first basic equation, adapted for the bladed regions of the compressor, is given by combining Equations 4, 5, and 6. If the input variable NEQN = 2, this equation is used by the computer program at computing stations that are at the edges of, or within, blade rows. (At leading edges, if the blade geometry is not given, the equation adapted for nonbladed regions is used.)

The second equation that we obtain is given by considering the forces $F_m$ and $F_n$ for nonbladed regions of the compressor. Our result differs slightly from that given by Wennerstrom in Reference 8, but he has agreed that the treatment here is correct. In the nonbladed regions, there can be no force normal to the streamline direction, nor can there by any change in angular momentum along a streamline. Then the forces become as follows:

$$F_m = -gJt\frac{dS}{dm} \tag{7}$$

$$F_n = 0 \tag{8}$$

The first basic equation, adapted for the nonbladed regions of the compressor, is given by combining Equations 4, 7, and 8. If the input variable NEQN = 2, this equation is used by the computer program at computing stations that are neither at the edges of, nor within, bladerows. If the input variable NEQN = 3, this equation is used at all computing stations.

For the second pair of equations, the basic momentum equation (Equation 4) is rewritten. The gradient $dV_m/dm$ is eliminated through use of the continuity equation. The rules of partial differentiation together with the geometric relationship

$$\frac{d\sin\phi}{dm} = \frac{\cos\phi}{r_c} \tag{9}$$

yield

$$\frac{dV_m}{dm} = \frac{\sin\phi}{\cos\gamma}\frac{dV_m}{dl} + (1-\tan\phi\tan\gamma)\frac{\partial V_z}{\partial z} + \frac{V_m\tan\phi}{r_c} - \frac{V_m}{\cos\gamma}\sin^2\phi\cos\phi\frac{d\tan\phi}{dl} \tag{10}$$

The term $\frac{\partial V_z}{\partial z}$ must now be eliminated, and the continuity equation is used for this. The continuity equation may be written (for axisymmetric flow) as

$$\frac{\partial}{\partial r}(\rho r\lambda V_r) + \frac{\partial}{\partial z}(\rho r\lambda V_z) = 0 \tag{11}$$

This may be expanded to give

$$-\frac{\partial V_z}{\partial z} = \frac{V_m}{\rho}\frac{d\rho}{dm} + \frac{V_m\sin\phi}{r} + \frac{V_m}{\lambda}\frac{d\lambda}{dm} + \frac{\sin\phi}{\cos\gamma(1-\tan\phi\tan\gamma)}\frac{dV_m}{dl} - \frac{\tan\gamma\tan\phi}{1-\tan\phi\tan\gamma}\frac{dV_m}{dm} + \frac{V_m\cos^3\phi}{\cos\gamma(1-\tan\phi\tan\gamma)}\frac{d\tan\phi}{dl} - \frac{\tan\gamma V_m}{(1-\tan\phi\tan\gamma)r_c} \tag{12}$$

Assuming the fluid to be a perfect gas, the meridional density gradient may be replaced using

$$\frac{1}{\rho}\frac{d\rho}{dm} = \frac{M_m^2}{V_m^2}\left(\frac{V_\theta^2 \sin\phi}{r} - \frac{W_\theta}{r}\frac{d(rV_\theta)}{dm} - V_m\frac{dV_m}{dm}\right) - \frac{J}{R}\frac{dS}{dm} \tag{13}$$

Note that the possibility of streamwise gradients of entropy and angular momentum is included. Equations 10, 12, and 13 are combined to give

$$\frac{dV_m}{dm}\left(1-M_m^2\right) = \frac{V_m}{r_c}\,\frac{\tan\phi+\tan\gamma}{1-\tan\phi\tan\gamma} - V_m\frac{d\tan\phi}{dl}\,\frac{\cos\phi}{\cos\gamma(1-\tan\phi\tan\gamma)}$$

$$-\frac{V_m\sin\phi}{r} - \frac{V_m}{\lambda}\frac{d\lambda}{dm} - \frac{M_m^2}{V_m}\left(\frac{V_\theta^2\sin\phi}{r} - \frac{W_\theta}{r}\frac{d}{dm}(rV_\theta)\right) + \frac{V_m J}{R}\frac{dS}{dm} \tag{14}$$

Equation 14 is used to eliminate $dV_m/dm$ from Equation 4 to give the second basic equation. For the bladed regions, Equations 5 and 6 again give $F_m$ and $F_n$, and the second basic equation adapted for the bladed regions of the compressor is obtained thus:

$$V_m\frac{dV_m}{dl} = \frac{V_m^2}{r_c}\,\frac{1-\cos^2(\phi+\gamma)M_m^2}{\cos(\phi+\gamma)(1-M_m^2)} - \frac{W_\theta}{r}\frac{d(rW_\theta)}{dl} - 2\omega W_\theta\cos\gamma$$

$$+ gJ\frac{dI}{dl} - gJT\frac{dS}{dl} + \frac{V_m}{r}\frac{d(rV_\theta)}{dm}\left\{\frac{M_m^2}{1-M_m^2}\tan\beta\sin(\phi+\gamma) - \tan\varepsilon\right\}$$

$$+ gJT\frac{dS}{dm}\left\{\sin(\phi+\gamma)\left(\cos^2\beta + \frac{\gamma M_m^2}{1-M_m^2}\right) - \tan\varepsilon\sin\beta\cos\beta\right\} \tag{15}$$

$$-\frac{V_m^2}{1-M_m^2}\tan(\phi+\gamma)\frac{d\phi}{dl} - \frac{V_m^2\sin(\phi+\gamma)}{1-M_m^2}\left\{\frac{\sin\phi}{r}\left(1+M_m^2\tan^2\alpha\right) + \frac{1}{\lambda}\frac{d\lambda}{dm}\right\}$$

If the input variable NEQN = 0, this equation is used by the computer program at computing stations that are at the edges of, or within, blade rows. (At leading edges, if the blade geometry is not given, the equation adapted for nonbladed regions, Equation 16, is used.)

For the nonbladed regions, the streamwise gradient of angular momentum in Equation 14 becomes zero, and $F_m$ and $F_n$ are given by Equations 7 and 8. Hence the second basic equation adapted for the nonbladed regions is obtained thus:

$$V_m \frac{dV_m}{dl} = \frac{V_m^2}{r_c}\,\frac{1 \quad \cos^2(\phi+\gamma)}{\cos(\phi+\gamma)(1-M_m^2)} - \frac{W_\theta}{r}\frac{d(rW_\theta)}{dl} - 2\omega W_\theta \cos\gamma$$

$$+ gJ\frac{dI}{dl} - gJt\frac{dS}{dl} + gJt\frac{dS}{dm}\;\frac{1-M_m^2(\gamma-1)}{1-M_m^2} - \frac{V_m^2\tan(\phi+\gamma)}{1-M_m^2} \times \tag{16}$$

$$\frac{d\phi}{dl} - \frac{V_m^2 \sin(\phi+\gamma)}{1-M_m^2}\left\{ \frac{\sin\phi}{r}\left(1+M_m^2\tan^2\alpha\right) + \frac{1}{\lambda}\frac{d\lambda}{dm} \right\}$$

If the input variable NEQN = 0, this equation is used by the computer program at computing stations that are neither at the edges of, nor within, bladerows. If the input variable NEQN = 1, this equation is used at all computing stations.

The remaining manipulations are those to write the equations in forms convenient for design or analysis-type calculations. For analysis calculations, when the relative flow angle ( $\beta$ ) is fixed, it is convenient to use a coordinate system rotating at blade speed. Then the rothalpy ($I$) is constant along streamlines through any one bladerow. Replacing $W_\theta$ by $V_m \tan\beta$ in Equation 4, and using Equations 5 and 6 for the forces $F_m$ and $F_n$, the analysis form of the first basic equation adapted for the bladed regions is obtained thus:

$$V_m \frac{dV_m}{dl}\sec^2\beta = \sin(\phi+\gamma) V_m \frac{dV_m}{dm} + \cos(\phi+\gamma)\frac{V_m^2}{r_c} - \frac{\tan\beta}{r}\frac{d(r\tan\beta)}{dl}$$

$$- 2\omega V_m \tan\beta\cos\gamma + gJ\frac{dI}{dl} - gJt\frac{dS}{dl} - \frac{\tan\varepsilon}{r}\frac{d(rV_\theta)}{dm} \tag{17}$$

$$+ gJt\frac{dS}{dm}\left\{ \sin(\phi+\gamma)\cos^2\beta - \tan\varepsilon\sin\beta\cos\beta \right\}$$

The analysis form of Equation 4 for nonbladed regions (which will only be used when blade force terms are deliberately omitted by setting NEQN to 3), is given by replacing $W_\theta$ by $V_m \tan\beta$ , and using Equations 7 and 8 for the forces $F_m$ and $F_n$ . The result is:

$$V_m \frac{dV_m}{dl}\sec^2\beta = \sin(\phi+\gamma) V_m \frac{dV_m}{dm} + \cos(\phi+\gamma)\frac{V_m^2}{r_c} - \frac{\tan\beta}{r}\frac{d(r\tan\beta)}{dl} \tag{18}$$

$$- 2\omega V_m \tan\beta\cos\gamma + gJ\frac{dI}{dl} - gJt\frac{dS}{dl} + \sin(\phi+\gamma)\, gJt\frac{dS}{dm}$$

For design-type calculations, because the absolute angular momentum is fixed, it is convenient to use stationary coordinates. Then for the bladed regions, instead of Equation 17, we obtain:

$$
\begin{aligned}
V_m \frac{dV_m}{dl} &= \sin(\phi+\gamma) V_m \frac{dV_m}{dm} + \cos(\phi+\gamma)\frac{V_m^2}{r_c} - \frac{V_\theta}{r}\frac{d(rV_\theta)}{dl} \\
&+ gJ\frac{dH}{dl} - gJt\frac{dS}{dl} - \frac{\tan\varepsilon\, V_m}{r}\frac{d(rV_\theta)}{dm} \\
&+ gJt\frac{dS}{dm}\left\{\sin(\phi+\gamma)\cos^2\beta - \tan\varepsilon\sin\beta\cos\beta\right\}
\end{aligned}
\tag{19}
$$

For the nonbladed regions, the design equation becomes:

$$
\begin{aligned}
V_m \frac{dV_m}{dl} &= \sin(\phi+\gamma) V_m \frac{dV_m}{dm} + \cos(\phi+\gamma)\frac{V_m^2}{r_c} - \frac{V_\theta}{r}\frac{d}{dl}(rV_\theta) \\
&+ gJ\frac{dH}{dl} - gJt\frac{dS}{dl} + gJt\frac{dS}{dm}\sin(\phi+\gamma)
\end{aligned}
\tag{20}
$$

Four corresponding equations are realized that are based upon the second basic momentum equation. The analysis form of Equation 15 (the equation for the bladed regions) is:

$$
\begin{aligned}
V_m \frac{dV_m}{dl}\sec^2\beta &= gJ\frac{dI}{dl} - gJt\frac{dS}{dl} - V_m^2\frac{\tan\beta}{r}\frac{d(r\tan\beta)}{dl} \\
&- 2\omega V_m\tan\beta\cos\gamma + gJt\frac{dS}{dm}\left\{\sin(\phi+\gamma)\left(\cos^2\beta + \frac{\gamma M_m^2}{1-M_m^2}\right)\right. \\
&\left. - \tan\varepsilon\sin\beta\cos\beta\right\} - \frac{V_m}{r}\frac{d(rV_\theta)}{dm}\left\{\tan\varepsilon - \frac{\sin(\phi+\gamma)M_m^2\tan\beta}{1-M_m^2}\right\} \\
&+ \frac{V_m^2}{1-M_m^2}\left\{\frac{1-\cos^2(\phi+\gamma)M_m^2}{\cos(\phi+\gamma)} - \tan(\phi+\gamma)\frac{d\phi}{dl} - \sin(\phi+\gamma)\left(\frac{\sin\phi}{r}(1+M_\theta^2)\right.\right. \\
&\left.\left. + \frac{1}{\lambda}\frac{d\lambda}{dm}\right)\right\}
\end{aligned}
\tag{21}
$$

For the nonbladed regions, the analysis form of Equation 16 is:

$$
\begin{aligned}
V_m \frac{dV_m}{dl} \sec^2\beta = {} & gJ \frac{dI}{dl} - gJt \frac{dS}{dl} - \frac{V_m^2 \tan\beta}{r} \frac{d}{dl}(r \tan\beta) \\
& - 2\omega V_m \tan\beta \cos\gamma + gJt \frac{dS}{dm} \cdot \frac{1 - M_m^2(\gamma - 1)}{1 - M_m^2} + \frac{V_m^2}{1 - M_m^2} \times \\
& \left\{ \frac{1 - \cos^2(\phi+\gamma) M_m^2}{\cos(\phi+\gamma)\, r_c} - \tan(\phi+\gamma) \frac{d\phi}{dl} - \sin(\phi+\gamma) \left( \frac{\sin\phi}{r} \left(1 + M_m^2 \tan\alpha\right) + \frac{1}{\lambda} \frac{d\lambda}{dm} \right) \right\}
\end{aligned}
\tag{22}
$$

(This equation will only be used when blade force terms are deliberately omitted by setting NEQN to 1.)

The design form of Equation 15 (in stationary coordinates, and for the bladed regions) is:

$$
\begin{aligned}
V_m \frac{dV_m}{dl} = {} & gJ \frac{dH}{dl} - gJt \frac{dS}{dl} - \frac{V_\theta}{r} \frac{d}{dl}(rV_\theta) + gJt \frac{dS}{dm} \left\{ \sin(\phi+\gamma) \left( \cos^2\beta + \frac{\gamma M_m^2}{1 - M_m^2} \right) \right. \\
& \left. - \tan\varepsilon \sin\beta \cos\beta \right\} - \frac{V_m}{r} \frac{d(rV_\theta)}{dm} \left\{ \tan\varepsilon - \frac{\sin(\phi+\gamma) M_m^2 \tan\beta}{1 - M_m^2} \right\} + \frac{V_m}{1 - M_m^2} \\
& \left\{ \frac{1 - \cos^2(\phi+\gamma) M_m^2}{\cos(\phi+\gamma)\, r_c} - \tan(\phi+\gamma) \frac{d\phi}{dl} - \sin(\phi+\gamma) \left( \frac{\sin\phi}{r} \left(1 + M_\theta^2\right) + \frac{1}{\lambda} \frac{d\lambda}{dm} \right) \right\}
\end{aligned}
$$

Finally, for the unbladed regions, where Equation 16 pertains, the design form is:

$$
\begin{aligned}
V_m \frac{dV_m}{dl} = {} & gJ \frac{dH}{dl} - gJt \frac{dS}{dl} - \frac{V_\theta}{r} \frac{d(rV_\theta)}{dl} + gJt \frac{dS}{dm} \sin(\phi+\gamma) \times \\
& \frac{1 - M_m^2(\gamma - 1)}{1 - M_m^2} + \frac{V_m^2}{1 - M_m^2} \left\{ \frac{1 - \cos^2(\phi+\gamma) M_m^2}{\cos(\phi+\gamma)\, r_c} - \tan(\phi+\gamma) \frac{d\phi}{dl} \right. \\
& \left. - \sin(\phi+\gamma) \left( \frac{\sin\phi}{r} \left(1 + M_\theta^2\right) + \frac{1}{\lambda} \frac{d\lambda}{dm} \right) \right\}
\end{aligned}
\tag{24}
$$

2. CONTINUITY EQUATION

The continuity equation is used at each computing station, and serves to fix the constant of integration in the momentum equation. The equation is:

$$
\begin{aligned}
W &= \int dW \\
&= \int \omega V_m \cos(\phi + \delta)\, 2\pi r\, (\lambda - B)\, d\ell
\end{aligned}
\tag{25}
$$

In order to determine on which branch of the continuity equation a velocity profile lies, and to be able to estimate by how much to change the velocity level to satisfy continuity, the rate of change of flow with midradius meridional velocity is required. If a perfect gas is assumed, then, for the design case (angular momentum fixed) it is convenient to write:

$$
\omega = \omega_T \left(1 - \frac{V_m^2 + V_\theta^2}{2gJC_pT}\right)^{\frac{1}{\gamma - 1}} \tag{26}
$$

Using this in Equation 25 and differentiating with respect to midradius meridional velocity, the result is:

$$
\frac{dW}{dV_{m_{mid}}} = \int \frac{dV_m}{dV_{m_{mid}}} \frac{1 - M_m^2}{V_m} dW \tag{27}
$$

It is assumed that:

$$
\frac{dV_m}{dV_{m_{mid}}} = \frac{V_m}{V_{m_{mid}}} \tag{28}
$$

Then the final result is:

$$\frac{dW}{dV_{m_{mid}}} = \frac{1}{V_{m_{mid}}} \int (1 - M_m^2)\, dW \tag{29}$$

For the analysis case, the relative flow angle and relative total density are constant, and we write:

$$\omega = \omega_{TR} \left(1 - \frac{V_m^2(1+\tan^2\beta)}{2gJC_pT_R}\right)^{\frac{1}{\gamma-1}} \tag{30}$$

Using this in Equation 25, and differentiating with respect to midradius meridional velocity, the result is:

$$\frac{dW}{dV_{m_{mid}}} = \int \frac{dV_m}{dV_{m_{mid}}} \frac{(1 - M_R^2)}{V_m}\, dW \tag{31}$$

For analysis type calculations, it is often important to evaluate Equation 31 accurately, and therefore Equation 28 is not assumed. Instead, the meridional velocity at any point on a computing station is represented by:

$$V_m(\ell) = V_{m_{mid}} + (\ell - \ell_{mid}) \frac{dV_m}{d\ell} \tag{32}$$

Hence:

$$\frac{dV_m}{dV_{m_{mid}}} = \frac{1}{1 - (\ell - \ell_{mid}) \frac{d}{dV_m}\left(\frac{dV_m}{d\ell}\right)} \tag{33}$$

The gradient $\dfrac{d}{dV_m}\left(\dfrac{dV_m}{dl}\right)$ is derived from Equation 21, 22, 17, or 18, depending upon whether the input variable NEQN = 0, 1, 2, or 3, respectively. In the cases where Equations 17 or 21 apply, the streamwise gradient of angular momentum is neglected. Thus from Equation 21, we obtain:

$$\frac{d}{dV_m}\left(\frac{dV_m}{dl}\right) = \cos^2\beta \left[ \frac{gJt\frac{dS}{dl} - gJ\frac{dI}{dl}}{V_m^2} - \frac{\tan\beta}{r}\frac{d}{dl}(n\tan\beta) + \frac{1}{1-M_m^2}\left\{ \frac{1-\cos^2(\phi+\gamma)M_m^2}{\cos(\phi+\gamma)\, r_c} - \tan(\phi+\gamma)\frac{d\phi}{dl} - \sin(\phi+\gamma)\left(\frac{\sin\phi}{r}(1-M_\theta^2)\right) + \frac{1}{\lambda}\frac{d\lambda}{dm}\right\} - gJt\frac{dS}{dm}\left(\frac{\sin(\phi+\gamma)\left(\cos^2\beta + \frac{\gamma M_m^2}{1-M_m^2}\right) - \tan\varepsilon\sin\beta\cos\beta}{V_m^2}\right)\right] \tag{34}$$

When Equation 22 applies, the result is the same except that the coefficient of $gJt\frac{dS}{dm}$ in Equation 34 is replaced by

$$\left(\frac{\sin(\phi-\gamma)\left(\frac{1-M_m^2(\gamma-1)}{1-M_m^2}\right)}{V_m^2}\right)$$

From Equation 17, we obtain

$$\frac{d}{dV_m}\left(\frac{dV_m}{dl}\right) = \cos^2\beta\left[\frac{\cos(\phi+\gamma)}{r_c} - \frac{\tan\beta}{r}\frac{d}{dl}n\tan\beta - \frac{1}{V_m^2}\left\{gJ\frac{dI}{dl} - gJt\frac{dS}{dl} + gJt\frac{dS}{dm}\left(\sin(\phi+\gamma)\cos^2\beta - \tan\varepsilon\sin\beta\cos\beta\right)\right\}\right] \tag{35}$$

When Equation 18 applies, the result is the same except that the coefficient of $gJt\frac{dS}{dm}$ in Equation 35 is replaced by:

$$\left( \sin(\phi+\gamma) \right)$$

So for analysis-type calculations, the gradient $dW/dV_{m\,mid}$ is given by Equations 31 and 33, with Equations 34 or 35 modified as appropriate.

3. FLUID PROPERTIES

As mentioned previously, the fluid properties are computed in the program for a perfect gas. The following basic relationships are used:

$$dh = t\,ds + \frac{1}{\rho}\,dp \tag{36}$$

$$h = C_p t \tag{37}$$

$$p = w R t \tag{38}$$

By arbitrarily defining the entropy as zero for a pressure and temperature of 1.0, the following relationships may be derived:

$$h = C_p \exp\left( S/C_p + R/JC_p \log_e p \right) \tag{39}$$

$$s = C_p \log_e (h/C_p) - R/J \log_e p \tag{40}$$

$$p = \exp\left( JC_p/R \log_e(h/C_p) - JS/R \right) \tag{41}$$

The three previous equations are used to relate enthalpies, pressure, and entropies. The following two relationships are also required:

$$\gamma = \frac{1}{\frac{R}{JC_p} - 1} \tag{42}$$

$$M^2 = \frac{C_p V^2}{\gamma g R h} \tag{43}$$

## 4. RELATIVE TOTAL PRESSURE LOSS COEFFICIENTS

Relative total pressure loss coefficients may be used in the input to specify, indirectly, the entropy rise in the flow, and are calculated for the output data. When the dynamic head specified to normalize the loss coefficient occurs at some location different to that where the total pressure is sought, the following definition applies:

$$P_r = P_r' - \bar{\omega}\left(P_{r_A} - p_A\right) \tag{44}$$

This gives the actual relative total pressure as the ideal value (computed from the station defined by the input variable NL1) less the pressure loss which is the product of the loss coefficient and the relative dynamic head (computed at the station specified by the input variable NL2). The isentropic relative total pressure $P_r'$ is given by the entropy at the station defined by NL1 and the relative total enthalpy. Basically, the same definition applies when the normalizing dynamic head is specified to occur at the location where the total pressure is sought (input variable NL2=0), but it is generally then more convenient to write the equation in the form

$$P_r = \frac{P_r'}{1 + \bar{\omega}(1 - p/P_r)} \tag{45}$$

## 5. ENTHALPY AND ROTHALPY, AND RELATED DEFINITIONS

The Euler turbomachine equation relates changes in angular momentum to change in total enthalpy thus

$$gJ(H_2 - H_1) = \omega(r_2 V_{\theta_2} - r_1 V_{\theta_1}) \tag{46}$$

Absolute and relative whirl velocities are related by

$$V_\theta = W_\theta + \omega r \tag{47}$$

Rothalpy is related to total enthalpy by

$$I = H - \omega r V_\theta / gJ \tag{48}$$

Relative total enthalpy is given by

$$H_r = H - \left(V_\theta^2 - (V_\theta - \omega r)^2\right)/2gJ \tag{49}$$

also

$$H_{r_2} - H_{r_1} = \omega^2 \left(r_2^2 - r_1^2\right)/2gJ \tag{50}$$

Static enthalpy is given by

$$h = H - V^2/2gJ \tag{51}$$

or

$$h = I + \left((\omega r)^2 - W^2\right)/2gJ \tag{52}$$

## 6. ANNULUS WALL BOUNDARY LAYERS

The program includes provision to compute blockage due to annulus wall boundary layers by a simple attached turbulent boundary layer method. The resulting blockage is applied as a uniformly distributed factor in the continuity equation. (It is a component of the term $B$ in Equation 25; the other component is the "wake blockage", specified in the input data by BBLOCK and BDIST.)

The boundary layer momentum thickness (on each wall) is calculated from

$$\theta = V_m^{-3.4}\, \nu^{0.2} \left(C_1 + 0.016 \int V_m^4\, dm\right)^{0.8} \tag{53}$$

Schlichting gives this equation (Reference 9, Equation 22.8) for a turbulent boundary layer on a flat plate with pressure gradient, except that the total velocity and flow length have been replaced by the meridional velocity component and meridional distance after the manner of Jansen, Reference 10. The equation is strictly valid for a constant shape factor (ratio of displacement to momentum thickness) of 1.4. In the program, any constant value may be used, and, for multistage machines, a value around 0.7 seems appropriate.

The constant $C_1$ is determined from the specified blockage at the inlet station which is assumed to occur as boundary layers of equal thickness on each wall. Then the integral is made from the inlet station to each station in the flow. If a hub does not exist in the machine at the inlet, all the inlet blockage is assigned to the casing. Integration on the hub then commences at the "bullet nose" where the boundary layer is assumed to have zero thickness.

## 7. LOSS COEFFICIENT RE-ESTIMATION

The loss coefficient re-estimation procedure, if enacted, consists of determining a diffusion loss coefficient and a shock loss coefficient, if applicable, and summing these to give the total loss coefficient. This is done for each streamline.

The diffusion loss coefficient is found by first calculating the diffusion factor from Equation 2. The corresponding loss parameter is then interpolated from each of the three loss parameter/diffusion factor curves specified to be used for the blade by |NEVAL|. These three curves give loss parameters for 10, 50, and 90 per cent of the radial blade height (measured at the blade trailing edge), and the value for the particular streamline radius is interpolated from the three values. Then, the diffusion loss coefficient is given by

$$\bar{\omega}_D = \bar{\omega}_{para}\, 2\sigma \sec\beta \tag{54}$$

The shock loss coefficient, if calculated (NDEL $\neq$ 0), is found from an average Mach number at the presumed strong shock location. A suction-surface Mach number is found by expanding (or compressing) the relative inlet Mach number through an expansion angle using the Prandtl-Meyer function.

$$PM = \frac{\gamma-1}{\gamma-1} \tan^{-1}\sqrt{\frac{\gamma-1}{\gamma-1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1} \tag{55}$$

The expansion angle is the sum of the value specified in the input (DELTA) and the incidence angle. If the blade geometry is not given at the blade leading edge (NDATA =0), the incidence is assumed to be zero. If the relative inlet Mach is less than unity, the expansion is calculated for an inlet value of unity. The shock Mach number is then taken as the mean of the relative inlet Mach number (or unity) and the suction surface Mach

number. If the relative inlet Mach number is less than unity, the shock Mach number is then multiplied by the inlet value. Should this final value be less than unity, the shock loss coefficient is set to zero. Otherwise, it is calculated from the pressure loss across a normal shock, using

$$\bar{\omega}_{shock} = \frac{\left(\frac{(\gamma+1)M_s^2}{(\gamma-1)M_s^2+2}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{\gamma+1}{2\gamma M_s^2-(\gamma-1)}\right)^{\frac{1}{\gamma-1}} - 1}{\left(1+\frac{\gamma-1}{2}M_{1r}^2\right)^{-\frac{\gamma}{\gamma-1}} - 1} \tag{56}$$

As mentioned above, the final loss coefficient is

$$\bar{\omega} = \bar{\omega}_D + \bar{\omega}_{shock} \tag{57}$$

8. OFF-DESIGN ANALYSIS SCHEME (NWORK =7)

The loss coefficient is defined in terms of a minimum value, $\bar{\omega}_{min}$ (input variable DATA2), a reference (or minimum loss) relative inlet angle, $\beta_{1REF}$, (DATA6), and "stalling" and "choking" inlet angles, $\beta_{1c}$ and $\beta_{1s}$ (DATA8 and DATA9). The loss coefficient at any inlet angle $\beta$ is then given by

$$\bar{\omega} = \bar{\omega}_{min}\left(1-\left(\frac{\beta-\beta_{1REF}}{\beta_{1s(orc)}-\beta_{1REF}}\right)^2\right) \tag{58}$$

where $\beta_{1s}$ or $\beta_{1c}$ is used depending upon whether $\beta$ is greater or less than $\beta_{1REF}$. A maximum value of 0.5 is imposed on the resulting loss coefficient. The relative outlet flow angle varies linearly from the reference value $\beta_{2REF}$ (DATA1) according to

$$\beta_2 = \beta_{2REF} + (\beta - \beta_{1REF})\frac{\partial\beta_2}{\partial\beta_1} \tag{59}$$

The gradient $\frac{\partial\beta_2}{\partial\beta_1}$ is given by the input variable DATA7.

## 9. BLADE GEOMETRY

For the purpose of determining relative flow angles, deviation angles, and incidence angles, blade angles are required to be known on the stream-surface plane.

When the input variable NWORK equals 6, or blade leading edge geometry is given, NWORK = 0 and NDATA > 0, the angle given in the input (as DATA1) is measured on the cylindrical plane. The corresponding value on the streamsurface is found from

$$\tan\beta = \left(\left(\tan\beta_{CYLINDRICAL}\left(1-\tan\gamma\tan\phi\right)-\tan\phi\tan\varepsilon_z\right)\cos\phi\right) \tag{60}$$

This equation is given in Reference 11. The angle $\varepsilon_z$ is related to the input blade lean angle by

$$\tan\varepsilon_z = \tan\varepsilon\sec\gamma \tag{61}$$

## 10. INTEGRATED PERFORMANCE

Machine performance is shown in the output in terms of pressure ratio and isentropic efficiency on each streamline, and also as "integrated" figures for the whole flow.

In order to determine the integrated values, the following definitions are made. At any station, the mean total enthalpy is given by

$$\bar{H} = \Sigma H.\delta W/W \tag{62}$$

The corresponding mean total pressure is given by

$$\bar{P} = \Sigma P.\delta W/W \tag{63}$$

From such values for the inlet station, a mean inlet entropy ( $\bar{S}_1$ ) is found. An isentropic outlet total enthalpy ( $\bar{H}_2'$) is then found as a function of the outlet mean total pressure ( $\bar{P}_2$ ) and $\bar{S}_1$ . The integrated isentropic efficiency is then given by

$$\eta = \frac{\bar{H}_2' - \bar{H}_1}{\bar{H}_2 - \bar{H}_1} \tag{64}$$

The inlet and outlet mean total pressures may also be used to determine an integrated total pressure ratio.

## 11. RADIAL TRANSFER OF BLADE WAKES

A phenomenon of the flow through typical compressor rotor blade rows is the high losses recorded for blade sections near the tip. This is illustrated in Figure 2.3 of Reference 6 where, for a given diffusion factor, rotor tip sections are shown to have a loss parameter from two to four times larger than other blade sections. In compressor design and analysis procedures, the traditional method has been to use different loss parameter /diffusion factor curves for different blade section locations. The analysis in this subsection is an attempt to account analytically for the phenomenon. It is postulated that the result seen comes about not from differences in the boundary layer development mechanism, but from radial transfers of blade boundary layer and wake flows, that is, secondary flows. These are caused by an imbalance of the radial forces on the slow-moving fluid particles, and the imbalance is the primary target of the following analysis. Because the resultant radial transfers are generally much greater for rotors than stators, the effect is usually only easily discernable there, but the analysis may be applied uniformly to rotors and stators. If the radial transfers are properly accounted for, the same wake generation model (loss coefficient correlation) may be used for all blade sections, provided it can be assumed that boundary layer development on any blade section is affected at most to a second order only by the radial transfers.

The approach followed is to determine the ratio of radial shear stress to axial shear stress on an element of the blade surface boundary layers. It is then postulated that the extent of the radial movement of the boundary layer/wake material is proportional to the force ratio. The radial force is found by considering the static pressure and centrifugal forces acting upon a small volume of blade surface boundary layer. This is then related to the shear stress at the blade surface by the boundary layer thickness. When the stress ratio is formed, the boundary layer thickness disappears from the final result.

The radial stress is found from consideration of the forces acting on a small volume of the boundary layer, which show the net radial force to be

$$F_r = \omega^2 r - \frac{1}{\rho}\frac{\partial p}{\partial r} \tag{65}$$

The small volume is in contact with the blade surface and, therefore, acts at the surface to produce a shear stress. If the boundary layer thickness is taken to be the conventional displacement thickness, then the shear stress is given by

$$\tau_r = \theta H \left( \omega^2 r \rho - \frac{\partial p}{\partial r} \right) \tag{66}$$

The axial stress is found as follows. For a turbulent boundary layer without pressure gradient, Schlichting (Reference 9, Equation 21.9) gives the momentum thickness as

$$\theta = .036\, x \left( \frac{U_\infty x}{\nu} \right)^{-1/5} \tag{67}$$

and the shear stress is

$$\tau_x = \rho U_\infty^2 \frac{\partial \theta}{\partial x} \tag{68}$$

At the blade trailing edge then, the axial stress is

$$\tau_x = \rho U_\infty^2 \frac{4}{5} \frac{\theta}{c} \tag{69}$$

The ratio of the stresses is

$$F'' = \frac{\tau_r}{\tau_x} = \frac{\theta H \left( \omega^2 r \rho - \frac{\partial p}{\partial r} \right)}{\rho U_\infty^2 \frac{4}{5} \frac{\theta}{c}} \tag{70}$$

If the shape factor is constant, we can write

$$F' = \frac{\left(\omega^2 r - \frac{1}{\rho}\frac{\partial p}{\partial r}\right)c}{U_\infty^2} \tag{7'}$$

It is now assumed that the tangent of the angle of the boundary layer/wake movement is proportional to $F'$. Thus a suitable axial dimension will yield the dimensional radial movement. For this dimension, the blade chord is taken. Hence:

$$F = K\left(\omega^2 r - \frac{1}{\rho}\frac{\partial p}{\partial r}\right)c^2/U_\infty^2 \tag{72}$$

For our purposes, the free stream velocity is the relative flow velocity. The constant $K$ remains to be determined from experimental data. In conjunction with the method of utilizing $F$ that is described below, a value of 2.0 appears to be approximately correct.

Equation 72 gives a radial movement distance at any radius, but some further assumptions are required before this gives a loss profile modified for the effects of radial transfers. Relative total pressure loss coefficient is approximately directly proportional to the thickness of the wake leaving the blade, and therefore the radial movement $F$ is directly applied to a two-dimensional loss coefficient distribution to produce the corresponding three-dimensional distribution. Inputs to the numerical procedure are loss coefficients presumed to be based upon blade element characteristics (and therefore designated two-dimensional) for each streamline, and $F$-values computed for each streamline. These two quantities are spline-fit interpolated at 150 points uniformly distributed between hub and casing. Each pair of values is taken to apply to an interval that lies about the corresponding radial location. Thus at radius $r_j$, there is a loss coefficient $\bar{\omega}_{2D}$ and a shift $F_j$ that apply to interval of width $b_j$, where

$$b_j = \left(r_{j+1} - r_{j-1}\right)/2.0 \tag{73}$$

In the case where $j = 1$ or 150, $b_j$ will be half the size of the other intervals. The loss coefficient from interval $b_j$ is redistributed according to $F_j$. It is assumed that the new distribution is a uniform division of the loss over the distance ( $b_j + |F_j|$ ), so that in that region the new loss coefficient attributable to $\bar{\omega}_{2D_j}$ is

$$\bar{\omega}_{3D} = \omega_{2Dj}\, b_j/(b_j + F_j) \tag{74}$$

The distributed loss coefficient $\bar{\omega}_{3D}$ is seen at all of the 150 radii that the movement $F_j$ crosses, including the starting point $r_j$. In the event that the movement $F_j$ prescribes a shift outside of the compressor annulus, $F_j$ is reduced so that the movement just reaches the annulus wall. Then $\omega_{3D}$ is proportionally larger than it would otherwise have been. This gives a "piling up" effect near a wall towards which there is boundary layer movement. The total loss coefficient at any radius is given by summing all of the $\bar{\omega}_{3D}$ components that apply to that radius. The last step is to determine loss coefficients for each of the streamlines that are being used in the basic flowfield calculation, and for which the original two-dimensional loss coefficients were given. To accomplish this, an eighth-order polynomial least-squares fit of the 150 $\bar{\omega}_{3D}$ values is made, and then loss coefficients at the desired radii are taken from the polynomial representation. The least-squares fit was introduced into the procedure because the 150-point curve exhibits some scatter and so cannot be used directly as input to a collating-curve interpolation scheme.

The procedure described above gives results that are, for engineering purposes, independent of the number of streamlines used to describe the flowfield. The selection of the number of points (150) at which the three-dimensional loss coefficients are derived, and the use of an eighth-order polynomial fit for the three dimensional loss coefficient distribution do have some effect on the results. As the number of points used to derive the loss coefficient distribution is reduced, the scatter of the resulting curve increases, and effectively less radial transfer occurs. This is because fewer of the shifts will be large enough to enter an adjacent accounting area. For larger numbers of points this effect diminishes, and there is generally little difference between the results for 100, 150, and 200 points. An eighth-order polynomial fit normally retains all the essential features of the 150-point distribution, without being unduly influenced by the scatter.

## 12. TURBULENT MIXING OF THE AXISYMMETRIC FLOW

Assumptions that are usually made in turbomachine flowfield analysis are that the flow is inviscid and nonconducting, so that there is no transfer of mass or energy across streamlines of the flow. The momentum equations derived earlier (in Section V. 1) incorporate these assumptions as well as the assumption that the flow is axisymmetric. In reality, the flow is viscous and turbulent in nature, and the effects of this are incorporated into the computation scheme.

The momentum equations are obtained by making the well-known assumption that the flow may be described by the Navier-Stokes equations for laminar flow, with the molecular kinematic viscosity replaced by an appropriate eddy viscosity. (See, for example, Reference 9.) For the purpose of analyzing the turbulence, the flow is assumed to be contained by frictionless walls. Thus the result will be the effects of mixing of the flow without regard for annulus wall boundary layers. In fact, the entire flow is assumed to be of a boundary layer nature. Thus the momentum equations, for axisymmetric, steady flow, and incorporating the boundary layer type flow assumption, which eliminates second derivatives of velocity in the streamwise direction, are as follows:

$$V_m \frac{dV_r}{dm} - \frac{V_\theta^2}{r} = \frac{1}{\rho}\frac{\partial p}{\partial r} + \epsilon\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_r}{\partial r}\right) - \frac{V_r}{r^2}\right) \tag{75}$$

$$\frac{V_m}{r}\frac{d}{dm}(rV_\theta) = \epsilon\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_\theta}{\partial r}\right) - \frac{V_\theta}{r^2}\right) \tag{76}$$

$$V_m \frac{dV_z}{dm} = \frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\epsilon}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) \tag{77}$$

It is now found convenient to consider the flow to be contained by a cylindrical annulus. By making this assumption, it follows that the radial velocity component must everywhere be very small compared to the other two velocity components, so that Equations 75, 76, and 77 may be written:

$$\frac{V_\theta^2}{r} = \frac{1}{\rho}\frac{\partial p}{\partial r} \tag{78}$$

$$V_z\frac{\partial V_\theta}{\partial z} = \frac{\epsilon}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial V_\theta}{\partial r}\right) - \frac{V_\theta}{r}\right) \tag{79}$$

$$V_z \frac{dV_z}{dz} = -\frac{1}{\rho}\frac{dp}{dz} + \frac{\epsilon}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) \tag{80}$$

It is interesting to note that Equation 78 is the Simple Radial Equilibrium Equation of turbomachinery, whereas Equations 79 and 80 retain viscous terms. This justifies a feature of the overall method that is introduced later: the assumption that even when turbulent mixing effects are incorporated into computation, the inviscid momentum equations derived in Section V.1 may be used to compute the velocity profiles at each computing station.

The boundary condition of frictionless walls may be expressed as:

$$\frac{\partial V_z}{\partial r} = \frac{\partial V_\theta}{\partial r} = 0 \quad \text{at the walls} \tag{81}$$

In order to determine the effect of turbulence upon the total enthalpy, an energy equation is required. This is obtained from the energy equation for laminar flow, neglecting conduction in the streamwise direction. Eddy viscosity is again substituted for molecular viscosity, and the turbulent Prandtl number is taken to be unity. Then the energy equation (for a cylindrical annulus) is:

$$V_z \frac{\partial H}{\partial z} = \frac{\epsilon}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right) \tag{82}$$

The adiabatic wall boundary condition may be expressed as

$$\frac{\partial h}{\partial r} = 0 \qquad \text{at the walls} \tag{83}$$

(This assumes that $t = t(h)$.)

Continuity is satisfied at each axial location by use of the relation:

$$W = \int V_z \rho \, dA \tag{84}$$

Equations 78, 79, 80, 82, and 84 with the boundary conditions given by Equations 81 and 83 describe the selected model of turbulent flow, and are used to determine changes in the distributions of entropy, total enthalpy, and angular momentum. The numerical procedure that is used is described in Section VI. 10. The eddy viscosity is presumed to be constant throughout the flowfield. This may be partially justified by considering that in the typical situation where an eddy viscosity model is used, (such as the mixing of a free jet, or the shear layer between two parallel flows), there is often a step change in velocity at the upstream boundary, whereas in the case of interest here the gradients in velocity are normally similar in magnitude throughout the flow. Application of the constant eddy viscosity model to cases with step changes in conditions at the upstream boundary could be expected to yield somewhat inaccurate profiles of conditions downstream, although the basic trends would be reproduced.

## SECTION VI

## NUMERICAL PROCEDURES FOR AERODYNAMIC ANALYSIS

### 1. INTEGRATION OF DESIGN MOMENTUM EQUATION

The design-case momentum equation (Equation 19, 20, 23, or 24, as appropriate) is integrated as a linear, first order equation in $V_m^2$ by assuming that only $V_m$ is unknown. Then the integrating factor technique may be applied to give:

$$V_{m_2}^2 = V_{m_1}^2 \exp\left(-A(\ell_2-\ell_1)\right) + \left(1-\exp\left(-A(\ell_2-\ell_1)\right)\right)\frac{B}{A} \tag{85}$$

where, in the case of Equation 19,

$$A = -2\left\{\frac{\sin(\phi+\gamma)}{V_m}\frac{dV_m}{dm} + \frac{\cos(\phi+\gamma)}{r_c}\right\} \tag{86}$$

and

$$B = 2\left\{gJ\frac{dH}{d\ell} - gJt\frac{dS}{d\ell} - \frac{V_\theta}{r}\frac{d(rV_\theta)}{d\ell} + gJt\frac{dS}{dm}\times \left(\sin(\phi+\gamma)\cos^2\beta - \tan\varepsilon\sin\beta\cos\beta\right) - \frac{V_m\tan\varepsilon}{r}\frac{d}{dm}(rV_\theta)\right\} \tag{87}$$

For Equation 20, $A$ remains unchanged, and the last two terms in $B$ are replaced by:

$$gJt\frac{dS}{dm}\sin(\phi+\gamma)$$

For Equation 23, the following definitions apply:

$$A = \frac{-2}{1-M_m^2}\left\{\frac{1-\cos^2(\phi+\gamma)M_m^2}{\cos(\phi+\gamma)\, r_c} - \tan(\phi+\gamma)\frac{d\phi}{d\ell} - \sin(\phi+\gamma)\left(\frac{\sin\phi}{r}\left(1+M_\theta^2\right) - \frac{1}{\lambda}\frac{d\lambda}{dm}\right)\right\} \tag{88}$$

$$B = 2\left\{ gJ\frac{dH}{dl} - gJt\frac{dS}{dl} - \frac{V_\theta}{r}\frac{d}{dl}(rV_\theta) + gJt\frac{dS}{dm} \times \right.$$

$$\left(\sin(\phi+\gamma)\left(\cos^2\beta + \frac{\gamma M_m^2}{1-M_m^2}\right) - \tan\varepsilon \sin\beta\cos\beta\right) - \tag{89}$$

$$\left.\frac{V_m}{r}\frac{d}{dm}(rV_\theta)\left(\tan\varepsilon - \frac{\sin(\phi+\gamma)M_m^2\tan\beta}{1-M_m^2}\right)\right\}$$

For Equation 24, $A$ remains unchanged, and the last two terms in $B$ are replaced by:

$$gJt\frac{dS}{dm}\ \sin(\phi+\gamma)\ \frac{1-M_m^2(\gamma-1)}{1-M_m^2}$$

Equation 85 fails if $A = 0$, but then the following relation applies

$$V_{m_2}^2 = V_{m_1}^2 + B(l_2 - l_1) \tag{90}$$

In the computer program, the switch from Equation 85 to Equation 90 is made when $|A| < 10^{-10}$

Equation 85 (or Equation 90) is applied to the interval between each adjacent pair of streamlines in turn. A single value of the parameters $A$ and $B$ is determined for each interval. Gradients in the direction of the computing station are found using:

$$\frac{dF}{dl} = \frac{F_2 - F_1}{l_2 - l_1} \tag{91}$$

Gradients in the streamline direction are found from, (for Station I),

$$\frac{dF}{dm} = \left(\frac{F_{I+1} - F_I}{m_{I+1} - m_I} + \frac{F_I - F_{I-1}}{m_I - m_{I-1}}\right)\Big/2 \tag{92}$$

At the first and last computing stations, and during the first pass of calculation, the streamwise gradients are assumed to be zero. A value is determined from Equation 92 for each streamline, and then an average value is used in each streamtube. This procedure is followed for each quantity that is given on a streamline rather than for a streamtube. The streamline slope angle $\phi$ is given, at Station I, by

$$\phi = \left( \tan^{-1}\left( \frac{r_{I+1} - r_I}{x_{I+1} - x_I} \right) + \tan^{-1}\left( \frac{r_I - r_{I-1}}{x_I - x_{I-1}} \right) \right) / 2 \tag{93}$$

At the first (last) computing station, $\phi$ is given by

$$\phi = \tan^{-1}\left( \frac{r_{I+1(-1)} - r_I}{x_{I+1(-1)} - r_I} \right) \tag{94}$$

The radius of curvature of the streamlines is assumed to be zero at the first and last stations and elsewhere is determined from:

$$\frac{1}{r_c} = \frac{\left( \tan^{-1}\left( \frac{r_{I+1} - r_I}{x_{I+1} - x_I} \right) - \tan^{-1}\left( \frac{r_I - r_{I-1}}{x_I - x_{I-1}} \right) \right) 2}{\left( m_{I+1} - m_{I-1} \right)} \tag{95}$$

The method of solution described above is based upon the assumption that the parameters A and B are fixed. In fact, they are dependent to a varying degree upon the meridional velocity, and so the determination of the meridional velocity is iterative. A meridional velocity profile is estimated, the parameters A and B are determined, and a new velocity profile is determined from the mid-streamline velocity and the A and B parameters. Some instability in this procedure was observed on occasion, and therefore a relaxation factor $R$ has been introduced into the procedure for the new estimate of the meridional velocities, thus:

$$V_{m_{new}} = V_{m_{old}} + R \left( V_{m_{calculated}} - V_{m_{old}} \right) \tag{96}$$

where

$$R = \exp\left( 11.52 \left| \frac{V_{m\,calculated}}{V_{m\,old}} - 1 \right| \right) \tag{97}$$

The nondimensional change in $V_m$ in Equation 97 is computed as an $\ell$-coordinate weighted average, and $R$ is not permitted to be less than 0.1.

Some limits are set on the values of $V_m$ that may be recorded for each streamline. A lower limit of 1.0, and an upper limit of twice the value on the mid-streamline are imposed, if necessary. For the case of Subroutine UD0308 only, there is also an upper limit equal to the local speed of sound.

When a new meridional velocity profile has been estimated by the above procedure, the entropy distribution is recalculated if losses are specified in such a way that the entropy is a function of meridional velocity. Then the continuity summations described separately below are made and, if continuity is not satisfied, or the resulting meridional velocity profile differs from that initially assumed, a new iteration is started. The maximum number of permitted iterations is controlled by the variable ITMAX, which is set (separately) to 20 in both Subroutine UD0308 and Subroutine UD0326.

## 2. INTEGRATION OF ANALYSIS MOMENTUM EQUATION

The analysis momentum equation (Equation 17, 18, 21, or 22, as appropriate) is nonlinear in $V_m$, and therefore a different scheme to that described above for the design case equation is used. All the gradients and streamline characteristics are computed as for the design case. Using an estimate of the average meridional velocity in a given streamtube, the gradient $dV_m^2/d\ell$ is computed from either Equation 17, 18, 21, or 22. This gradient is then used to step to the next streamline using:

$$V_{m_2}^{\,2} = V_{m_1}^{\,2} + (\ell_2 - \ell_1)\frac{dV_m^{\,2}}{d\ell} \tag{98}$$

The velocity determined by Equation 98 is not permitted to be less than 1.0 nor greater than three times the mid-streamline value. Because the gradient depends upon the meridional velocity, the velocity determination by this means is iterative, and up to 10 loops are allowed.

When a new meridional velocity profile has been established by the above procedure, the entropy distribution is recalculated if losses are specified in such a way that the entropy is a function of meridional velocity. Then the continuity summations described separately below are made and, if continuity is not satisfied, or the resulting meridional velocity profile

differs from that initially assumed, a new iteration is started. The maximum number of permitted iterations is controlled by the variable ITMAX, which is set (separately) to 20 in both Subroutine UD0308 and Subroutine UD0326.

## 3. SATISFACTION OF CONTINUITY EQUATION

The continuity equation (Equation 25) is required to be satisfied at each computing station simultaneously with the appropriate momentum equation. Accordingly, the following procedure is incorporated into the up to 20 iterations that may be performed to satisfy the momentum equation.

The flow rate is determined from Equation 25 using trapezoidal integration, and the rate of change of flow with mid-streamline meridional velocity is similarly calculated from Equation 29 or 33, as appropriate. Several courses of action may follow.

If the solution type is that desired (that is $dW/dV_m$ is positive when a subsonic solution is specified or negative when a supersonic solution is specified), then the meridional velocity increment to satisfy continuity is estimated from

$$\Delta V_m = \frac{W_{specified} - W_{calculated}}{dW/dV_m} \tag{99}$$

The velocity increment is limited to ±10% of the previous mid-streamline value. Also, if the flow rate is less than the desired value, and a valid solution to the momentum equation was previously computed, the mid-streamline meridional velocity is noted, as it is a value below which the correct value cannot ' e for the subsonic case, or above which the correct value cannot lie for the supersonic case.

If the solution type is not that desired, the velocity increment is arbitrarily set to plus or minus 10% of the mid-streamline value in an attempt to switch to the supersonic or subsonic case, respectively. Also, if a valid solution to the momentum equation was previously computed, the mid-streamline velocity is noted because, for example, when a subsonic solution is desired, no meridional velocity should be evaluated which is known to produce a supersonic solution.

When an analysis-type momentum equation is being used only, a check is now made on the velocity increment that has been determined by either of the methods described above. Suppose that a subsonic solution is desired. (The logic is "symmetrical" for the case of a desired supersonic solution.) The mid-stream line meridional velocity that would be

given by modifying the previous value by the calculated increment is checked to see if it would be higher than the lowest value which has been shown to imply a supersonic solution, $V_{m\,super}$ . If so, a new increment is calculated. The new increment is given by

$$\Delta V_m = \left(V_{m\,super} - V_{m\,old}\right)/2 \tag{100}$$

Thus, the new velocity will be within the range of possible values, and will generally refine the previously established limits for the range.

A similar check is made with respect to the minimum plausible velocity, that is $V_{m\,sub}$ , the highest velocity shown to imply a subsonic solution at a flow rate lower than desired.

The meridional velocities on each streamline are incremented by the amount derived as described above, and a lower limit of 1.0 is imposed on the value produced.

If Subroutine UD0308 is being used (input variable NEQN = 0 or 1), an upper limit equal to the local sound speed is also imposed upon the meridional velocity at each streamline. In this case, the maximum mid-streamline meridional velocity allowable for a supersonic solution ( $V_{m\,super}$ ) is set equal to (the sound speed x 1.05) initially.

When Subroutine UD0326 is being used (input variable NEQN = 2 or 3) $V_{m\,super}$ is initialized to 2500.0. In both cases, the minimum permitted velocity for a subsonic solution is initialized to zero.

A check is made on the convergence of the momentum and continuity equations, and, if these do not both converge, a new iteration is started, subject to the maximum number of iterations permitted by the variable ITMAX. If a new iteration is started, the entropy distribution is recalculated if it is a function of meridional velocity, and then the momentum equation integration procedure is entered as described above.

## 4. STREAMLINE RELOCATION ITERATION

The positions of the streamline are determined iteratively using the condition that a constant fraction of the total flow should lie in each streamtube. The flow distribution used to revise the streamline coordinate is a function of the streamline characteristics, and, therefore, the calculation is potentially unstable. A relaxation factor is derived in the following analysis, which is based upon a similar analysis performed by Wilkinson for two-dimensional flow, Reference 12.

Consider a swirling flow in a cylindrical annulus, with no enthalpy or entropy gradients. Computing stations are uniformly spaced $\Delta x$ apart, and are radial. Equation 20 may be applied, and simplified to give

$$\frac{1}{2}\frac{dV_m^2}{dr} = \frac{V_m^2}{r_c} - \frac{V_\theta}{r}\frac{d}{dr}(rV_\theta) \qquad (101)$$

By assuming only small deviations from midradius conditions, we can approximate Equation 101 by

$$\frac{dV_m}{dr} = \frac{V_{m_{mid}}}{r_c} - K \qquad (102)$$

If we introduce an error in the streamline radius at one station only, then the corresponding radius of curvature is

$$r_c = -\frac{\Delta x^2}{2E} \qquad (103)$$

Combining Equations 102 and 103 gives

$$\frac{dV_m}{dr} = V_{m_{mid}}\frac{2E}{\Delta x^2} - K \qquad (104)$$

or

$$V_m(r) = V_{m_{mid}} - \left(\frac{2EV_{m_{mid}}}{\Delta x^2} - K\right)(r - r_{mid}) \qquad (105)$$

Mass flux distribution is proportional to $\rho V_m$, and again, we approximate the distribution, using

$$\rho V_m = (\rho V_m)_{mid} + (V_m - V_{m_{mid}})Q\rho_{mid} \qquad (106)$$

and

$$Q = \left(\frac{1}{\rho}\frac{d}{dV_m}(\rho V_m)\right)_{mid} \tag{107}$$

For a high radius ratio, the flow between radii $r_1$ and $r_2$ is proportional to $\int_{r_1}^{r_2} \rho V_m \, dr$ and from the previous equations

$$\int_{r_1}^{r_2} \rho V_m \, dr = (\rho V_m)_{mid}\left\{(r_2 - r_1) + \frac{1}{2}\left(\frac{2EC}{\Delta x^2} + K\right)\left((r_2 - r_{mid})^2 - (r_1 - r_{mid})^2\right)\right\} \tag{108}$$

To determine the effect of the error, we compare the predicted location of the streamline that divides the flow into two equal parts with its correction location, that is when $E = 0$ The predicted location, $r_x$ , may be found from the following equation

$$.5 = \int_{r_{hub}}^{r_x} \rho V_m \, dr \bigg/ \int_{r_{hub}}^{r_{case}} \rho V_m \, dr \tag{109}$$

Using Equation 108, and assuming

$$r_x - r_{hub} \approx r_{mid} - r_{hub} \tag{110}$$

this yields

$$r_x = r_{mid} - \left(\frac{QE}{\Delta x^2} + \frac{K}{2 V_{m_{mid}}}\right)\frac{\Delta R^2}{4} \tag{111}$$

where

$$\Delta R = r_{case} - r_{hub} \tag{112}$$

The correction location, $r_o$ , is similarly found to be

$$r_o = r_{mid} - \frac{K \Delta R^2}{8 V_{m_{mid}}} \tag{113}$$

Thus, the shift in streamline position is

$$r_x - (r_o + E) = -E\left(1 + \frac{Q \Delta R^2}{4 \Delta x^2}\right) \tag{114}$$

The shift to correct the error $E$ should be $-E$ , so that the correspon-ding relaxation factor $R$ , is

$$R = \frac{1}{1 + \frac{Q}{4}\left(\frac{\Delta R}{\Delta x}\right)^2} \tag{115}$$

The case considered, that is where the error $E$ is imposed at one station only, is one extreme. It produces the largest streamline curvatures, and, without a relaxation factor, the largest shifts in streamline radius from one pass to the next. The other extreme is when the error is imposed at all stations, producing no curvature. In this case, any damping (that is, a relaxation factor of less than unity), will cause the streamline shift to be underestimated. An optimum relaxation factor $F$ is defined as one which reduces the error to the same fraction for both cases. Noting the sign change occurring between the two cases, we obtain

$$F - 1 = 1 - F\left(1 + \frac{Q}{4}\left(\frac{\Delta R}{\Delta x}\right)^2\right) \tag{116}$$

so that

$$F = \frac{1}{1 + \frac{Q}{8}\left(\frac{\Delta R}{\Delta x}\right)^2} \tag{117}$$

The remaining term $Q$ is determined by assuming the fluid to be a perfect gas. Then

$$\rho V_m = \rho_T V_m \left(1 - \frac{V_m^2 + V_\theta^2}{2gJGT}\right)^{\frac{1}{\gamma-1}} \tag{118}$$

and

$$Q = \left(\frac{1}{\rho} \frac{d}{dV_m}(\rho V_m)\right)_{mid} = 1 - M_m^2 \tag{119}$$

For the case when the relative flow angle is specified, the analysis starts from Equation 18, and, following the same steps as for the case above, the final result is

$$F' = \frac{1}{1 + \frac{Q'}{8}\cos^2\beta\left(\frac{\Delta l}{\Delta x}\right)^2} \tag{120}$$

and

$$Q' = 1 - M_{r\,mid}^2 \tag{121}$$

In the program, the length $\Delta l$ is taken as the computing station length and the length $\Delta x$ is taken as the meridional distance at mid-annulus to the nearer of the two adjacent computing stations. The constant 8 in Equations 117 and 120 has been found fron experience to often need reducing to 6 to ensure stability. It is input as the variable RCONST. The Mach number squared that appears in Equations 119 and 121 is limited in the program to the value input as XMMAX, and 0.6 has been found to be generally satisfactory. The relaxation factor finally determined is applied to the streamline coordinate $l$, thus

$$l_{new} = l_{old} + F^{(')}\left(l_{calculated} - l_{old}\right) \tag{122}$$

## 5. CONVERGENCE CRITERIA

The preceding four subsections have all described iterative procedures involved in the overall determination of a solution. The convergence of each

of these procedures is measured by means of a tolerance, applied as described below, and related to a single input variable, TOLNCE. The normal value for this is 0.001.

When the meridional velocity at a station is determined by integration of the analysis form of the momentum equation, the velocity at any streamline is iteratively determined in a stepping procedure from the adjacent streamline that is nearer to the mid-streamline. This iteration is considered converged if the calculated velocity repeats to within a tolerance of TOLNCE/5.0, applied nondimensionally. Thus, if TOLNCE = 0.001, two successive meridional velocities must be within one part in 5000.

Following adjustment of the velocity level on all stream lines, a check is made to see if both continuity and momentum are satisfied. The tolerance used is TOLNCE/5.0, unless a loss coefficient re-estimation is specified at the station (NEVAL > 0), when the tolerance is TOLNCE/10.0. The continuity check is that the calculated flow must equal the specified value to within the stated tolerance, applied nondimensionally. Then the momentum equation solution is checked by requiring that the velocities equal those estimated to within the same tolerance, applied nondimensionally.

If loss coefficient re-estimation is specified at a computing station, an extra iteration for the loss coefficients is involved. Convergence of this iteration is measured by comparing the estimated meridional velocities with those calculated, and the tolerance applied is TOLNCE/5.0. With LITER set to say 5, this iteration will probably not converge to within the above stated tolerance during the first few passes through the program.

Overall convergence of the solution is determined by two checks that are made at the end of each pass. The meridional velocities at each mesh point must repeat to within a tolerance of TOLNCE, applied nondimensionally. Also, the proportion of the total flow that passes between the hub and each streamline unit be within TOLNCE of the desired value at every computing station. For example, if some streamline should have 0.5 of the flow between it and the hub, and TOLNCE = 0.001, the acceptable range is from 0.499 to 0.501.

## 6. PRANDTL-MEYER FUNCTION

The Prandtl-Meyer function is given by Equation 55, and in the course of re-estimating the loss coefficients, if this is specified to be done, it is desired to determine the Mach number corresponding to a given angle, $\Theta$. This is done iteratively using Newton's method, with the quantity $\sqrt{1-M^2}$ being sought (rather than $M$). An initial estimate, $F_1$, of $\sqrt{1-M^2}$ is made equal to $\sqrt{1-M_1^2}+0.1$ where $M_1$ is the Mach number before the expansion. Then the new estimate, $F_2$, is given by

$$F_2 = F_1 - \frac{\left(\frac{\gamma+1}{\gamma-1} + F_1^2\right)\left(1 + F_1^2\right)}{F_1^2\left(\frac{\gamma+1}{\gamma-1} - 1\right)} \left\{ \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\left(F_1 \Big/ \sqrt{\frac{\gamma-1}{\gamma+1}}\right) - \tan^{-1} F_1 - \theta \right\} \quad (123)$$

This is repeated until $F_1$ and $F_2$ are within 0.000001 (subject to a maximum number of iterations of 20).

## 7. STAGNATION POINT PROVISION

If the "hub" radius at the inlet station is zero, a stagnation point is incorporated into the solution at the last station for which the "hub" radius is zero. This must occur upstream of any blade rows, and the whirl velocity at zero radius must be zero. The stagnation point is handled by imposing the zero meridional velocity at that point, and only integrating the momentum equation inwards as far as the second streamline. The continuity summation includes the innermost streamline in the normal manner. A finite curvature will be shown in the results at the stagnation point, but this is of no consequence because the momentum equation is not computed in the innermost streamtube.

## 8. INTERPOLATION

The various tabulated data input to the program is interpolated at the streamline by the method specified by the input variable NTERP. Two alternatives are available. If NTERP = 1, linear point-to-point interpolation (or extrapolation) is performed. This is not to be preferred generally, assuming the data points define a smooth curve.

If NTERP = 0, spline-fit interpolation is performed. The spline curve consists of a series of algebraic cubics, one equation applying between each adjacent pair of data-points. The coefficients are determined such that the resultant curve passes through each point, and has continuous first and second derivatives. Various boundary conditions may be imposed at each end of the curve; in this case the second derivative at the end point is set to zero.

## 9. DEVIATION ANGLE DETERMINATION FOR ANALYTIC MEANLINE BLADE SECTION

The determination of the deviation angles for analytic meanline blades is performed in Subroutine UD0319. A description of the procedure used is included in this section. The subroutine is only entered if the input variable NAERO = 1.

The trailing edge deviation is calculated from

$$\delta_{te} = \delta_{o_{10}} K_{sh} K_{tc} + \frac{\theta}{\sigma^{b}}\left(m + 0.5\left(a/c - 0.5\right)\right) + \delta_{ADD} \tag{124}$$

The parameters $K_{sh}$, $a/c$, and $\delta_{ADD}$ are given in the input as variables XKSHPE, AC, and DELTAD, respectively. The camber ( $\theta$ ) and solidity ( $\sigma$ ) are derived from the basic determination of the blade sections. The remaining parameters, $\delta_{o_{10}}$, $K_{tc}$, $b$, and $m$, are found by interpolation from data given in Figures 161, 172, 164, and 195 of Reference 6, respectively. Equation 124 is based upon Equations 269 and 271 of Reference 6, except that, when a parameter is shown in Reference 6 to be a function of relative air inlet angle, blade inlet angle is substituted in this program.

For locations, that is, computing stations, within a blade row, the deviation is determined from

$$\delta = \delta_{te}\left(\delta/\delta_{te}\right) \tag{125}$$

The fraction $\delta/\delta_{te}$ is interpolated from the input variables DVFRAC and DM as a function of the location on the meridional chordline and, if NRAD > 1, as a function of streamsurface radius at the trailing edge.

The "station control card", blade speed, blade geometry, loss values (interpolated at the appropriate radii), and deviation angles are output onto the file specified by unit LOG5 for each computing station within the blade, or at the blade trailing edge, in the format required by the aerodynamic section. The station control card has NDATA set equal to the number of streamsurfaces on which blade sections are produced, NWORK = 6, and NDIMEN = 0. The remaining variables are set according to the values in the input data for that station.

At the blade leading edge, no deviation calculation is required, but a "station control card", a speed of zero, and blade geometry are output onto the file specified by unit LOG5 if the input variable IFANGS for the leading edge station is 1. The station control card has NDATA set equal to the number of streamsurfaces on which blade sections are produced, and all remaining variables on the control card are set to zero, except NOUT1, NOUT2, and NOUT3, which are reproduced as read in.

## 10. TURBULENT MIXING CALCULATION PROCEDURE

The equations derived in Section V. 12 describe the changes in the distribution of conditions along "streamlines" of an axisymmetric, turbulent flow in a cylindrical annulus. In Section V. 1, equations were derived for the stationwise variation in meridional velocity of an axisymmetric, inviscid flow in an arbitrary annulus. These show the meridional velocity gradient to be a function of the distributions of entropy, total enthalpy, and angular momentum (as well as the streamline characteristics). In order to incorporate the mixing calculations into the overall computations, first the turbulent flow equations are used to determine modifications to the stationwise distributions of entropy, total enthalpy, and angular momentum. Then, the modified distributions are used in an inviscid momentum equation to compute the meridional velocity profile. This method is used for the interval between each successive pair of computing stations from the first to the last, for each pass through the iterative procedure to determine the streamline locations, and hence the final solution. On a step-by-step basis, the computation proceeds thus:

1) At Station 1, an inviscid momentum equation gives the meridional velocity distribution.

2) The turbulent flow equations are applied for the interval Station 1 to Station 2 to give the distributions of entropy, total enthalpy, and angular momentum at Station 2.

3) If a blade-row lies between Stations 1 and 2, the distributions of entropy, total enthalpy, and angular momentum at Station 2 are redetermined, using the modified distributions determined in step 2 as the inlet conditions. Thus the effects of the blade-row (if one exists) are superimposed upon the mixing calculation.

4) An inviscid momentum equation is used to determine the meridional velocity distribution at Station 2.

5) Steps 2, 3, and 4 are repeated for the intervals Station 3 to 4, 4 to 5, and so on, to the last computing station.

6) A new streamline pattern estimate now exists, based upon the meridional velocity profiles computed for each station. Unless the overall solution is converged, control returns to Step 1 for another pass.

The application of the turbulent flow equations, Step 2 above, is made as follows. The interval of compressor annulus is approximated by a cylindrical annulus, for which the hub and tip radii are the mean hub and

tip radii of the segment of the compressor annulus, and for which the length is the mean length of the segment of compressor annulus. The evolution of conditions at the downstream boundary from those at the upstream boundary is made by stepping downstream through a number of equal increments. The method by means of which the number of increments is chosen is described below. Each increment may be considered to have an upstream boundary and a downstream boundary. Conditions at the first upstream boundary are those at the upstream boundary of the whole mixing interval. Conditions at the downstream boundary of an intermediate step become the upstream boundary conditions for the next step, until finally for the last step, the downstream boundary conditions are those sought for the downstream boundary of the mixing interval. Inputs to the calculation are, at the upstream boundary, the mid-streamline static pressure, the mid-streamline meridional velocity (both as computed previously by the inviscid equations), the radial distributions of entropy, total enthalpy, and angular momentum, and the eddy viscosity. For each mixing step, the calculation is made as follows:

1) The radial component equation (Equation 78) is integrated to give the static pressure distribution at the upstream boundary. Equation 78 is written in the following form:

$$\Delta p = \frac{V_\theta^2}{r} \rho \Delta r \tag{126}$$

   This equation is used to step from streamline to streamline, mean values of the quantities involved being used in each radial interval. As the density distribution is not known, this must be determined itervatively from the static pressures and the entropy.

2) For each streamline, the static pressure and entropy give the static enthalpy. The total and static enthalpies give the total velocity. The total velocity and tangential velocity component give the axial velocity component.

3) For the first step only, a continuity summation is made to determine the flow rate.

4) The energy equation (Equation 82) is applied on each streamline to give the total enthalpy distribution at the downstream boundary. Equation 82 is written as follows:

$$\Delta H = \frac{\Delta z}{V_z} \frac{\xi}{r} \frac{\partial}{\partial r}\left(r \frac{\partial h}{\partial r}\right) \tag{127}$$

The method of obtaining all radial derivatives is described below.

5) The tangential component equation (Equation 79) is used on each streamline to give the downstream tangential velocity component distribution. Equation 79 is written:

$$\Delta V_\theta = \frac{\Delta z}{V_z} \frac{\xi}{r} \left( \frac{\partial}{\partial r}\left(r \frac{\partial V_\theta}{\partial r}\right) - \frac{V_\theta}{r} \right) \tag{128}$$

6) The mid-streamline, downstream static pressure is estimated (as being equal to the upstream value).

7) The downstream radial distribution of static pressure is determined as was the upstream distribution in Step 1.

8) The axial component equation (Equation 80) is applied on each streamline for the downstream axial velocity distribution. Equation 80 is written:

$$\Delta V_z = \frac{1}{V_z} \left( \frac{\xi \, \Delta z}{r} \frac{\partial}{\partial r}\left(r \frac{\partial V_z}{\partial r}\right) - \frac{\Delta p}{\rho} \right) \tag{129}$$

9) For each streamline, total enthalpy, tangential and axial velocities yield static enthalpy. Static enthalpy and static pressure give density. A summation is made for the flow rate at the downstream boundary.

10) If the flow rate computed in Step 9 differs from that established in Step 3, the mid-streamline static pressure is re-estimated, and control returns to Step 7. (The method of re-estimating the static pressure is given below.) Otherwise, control passes to Step 11.

11) Entropy at the downstream boundary is given by static enthalpy and static pressure. Downstream boundary conditions become upstream boundary conditions for the next step, and control returns to Step 1.

Radial derivatives of the form $\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right)$ occur in Equations 127, 128, and 129. For streamlines away from the walls, the following method is used:

$$\left(\frac{\partial}{\partial r} r\left(\frac{\partial F}{\partial r}\right)\right)_J = \frac{\dfrac{(r_J + r_{J+1})(F_{J+1} - F_J)}{r_{J+1} - r_J} - \dfrac{(r_J + r_{J-1})(F_J - F_{J-1})}{r_J - r_{J-1}}}{r_{J+1} - r_{J-1}} \tag{130}$$

Values near a wall must reflect the boundary condition: $\frac{\partial F}{\partial r} = 0$ at the wall. If the streamlines are numbered 1, 2, 3, ... starting at the wall, then

$$\left(\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right)\right)_2 = \frac{(r_3 + r_2)(F_3 - F_2)}{(r_3 - r_2)(r_3 - r_1)} \tag{131}$$

A value for streamline 1 is obtained by linearly extrapolating the values for streamline 3 and 2 to the radius at the wall. The value at streamline 3 is given by the general formula, Equation 130.

The number of steps into which each interval must be divided is controlled by the stability of the three equations used to step in the axial direction. Consider the energy equation, and a linear variation of enthalpy with radius. The effect of mixing will be to reduce the enthalpy gradient, but not to reverse the sign thereof. If an excessively large axial step is made, the sign of the gradient will change, and so the limit for stability is when the gradient is just eliminated. Writing the energy equation in approximate form:

$$\Delta h = \frac{\Delta z}{V_z} \varepsilon \frac{\partial^2 h}{\partial r^2} \tag{132}$$

The critical streamlines are those adjacent to the walls. If the streamlines are numbered 1, 2, 3, .... starting at the wall, and equispaced, then, from Equations 130 and 131 (and for a linear enthalpy distribution):

$$\Delta h_{\substack{\text{axial direction}\\ \text{streamline 2}}} = \frac{\Delta z}{V_z} \xi \frac{\Delta h}{\Delta r^2} \qquad (133)$$

$$\Delta h_{\substack{\text{axial direction}\\ \text{streamline 1}}} = \frac{\Delta z}{V_z} 2\xi \frac{\Delta h}{\Delta r^2} \qquad (134)$$

where $\Delta h$ is the change in enthalpy between streamlines and $\Delta r$ is the streamline spacing.

The limit for stability is when:

$$h_1 + \Delta h_{axial,1} = h_2 + \Delta h_{axial,2}$$

or

$$\Delta h_{axial_1} - \Delta h_{axial_2} = \Delta h$$

and from Equations 133 and 134

$$\Delta z \geqslant \frac{V_z \Delta r^2}{\xi} \qquad (135)$$

The same analysis may be applied to the tangential and axial component equations, with the same result. In practice, the limit of $\Delta z$ has been found to be slightly more than half the value given by Equation 135. The program selects the number of steps to be four times that given by Equation 135, but even this will often lead to only one step being required for each mixing interval.

The re-estimation of the mid-streamline static pressure to satisfy continuity at the downstream boundary of each mixing step (Step 9 in the procedures above), is accomplished by determining the value of change of flow with static pressure for the one-dimensional flow of a perfect gas. Then the required static pressure change is found to be:

$$\Delta p = \frac{\left(W_{desired} - W_{calculated}\right) p \, M_z^2}{W\left(1 - M_z^2\right)} \tag{136}$$

where $M_z$ is the ratio of axial velocity to local sound speed.

## SECTION VII

## PROGRAM OPERATION AND STRUCTURE

### 1. IMPLEMENTATION

The program is basically written in USA Standard FORTRAN and should therefore be compatible with all current medium-to-large computing systems, except as noted below. All development running has been done on a Control Data Corporation 6600 computer at Wright-Patterson Air Force Base. The operating system was changed from Scope 3.3 to Scope 3.4 during the program development period. The operating system includes CALCOMP software and hardware for on-line precision plotting.

The program is loaded in three principal overlays by a small resident main program, and the demands of the CDC Scope overlay scheme are incorporated into the coding. Under Scope 3.4, the central memory requirements for the three overlays are approximately as follows: the aerodynamic section, 115K; the analytic meanline blade section, 125K; and the arbitrary meanline blade section, 105K. These requirements would be reduced about 10K under the more widely used Scope 3.3 system. The aerodynamic section is divided into a principal loading and two alternative loadings. The alternative loads are Subroutines UD0308 and UD0326, which handle two alternative momentum equation formulations selected by the input variable NEQN. In order to compile and run the program under systems other than CDC Scope, the overlay-related calls will need to be modified. These are:

1) Card $MN$.24 in Program UD0300 (the main program) which calls the analytic meanline section, cards $AN$.2 and $AN$.3.

2) Card $MN$.27 in the main program,which calls the aerodynamic section, cards $AR$.2 and $AR$.3.

3) Card $MN$.31 in the main program, which calls the arbitrary meanline section, cards $AB$.2 and $AB$.3.

4) Card $AR$.229 in Program UD03AR, which calls Program UD0308, cards $08$.2 and $08$.3.

5) Card $AR$.231 in Program UD03AR, which calls Program UD0326, cards $26$.2 and $26$.3.

With the exception of the main program, each of the programs referred to above is, in conventional FORTRAN terms, a Subroutine. The PROGRAM

statement attached to the main program, Card $MN$. 2, is peculiar to the CDC Scope system, and defines the following file usage by the program:

| CDC File Name | FORTRAN Variable Name | Value | Description |
|---|---|---|---|
| INPUT | LOG1 | 1 | Card reader |
| OUTPUT | LOG2 | 2 | Line printer |
| PUNCH | LOG3 | 3 | Card punch |
| FILE2 | LOG5 | 5 | Scratch file |
| FILE3 | LOG6 | 6 | Scratch file |
| PLOT | - | - | CALCOMP plot file |

The numerical values are assigned to the FORTRAN variables in the main program starting with card $MN$. 8. (LOG4 is redundant.) LOG1 and LOG2 are also defined in Subroutine UDG1 on cards $GN$. 4 and $GN$. 5.

Calls are incorporated into the coding to subroutines that are assumed to be available as part of a CALCOMP plotting package. All the plotting is only performed optionally so that the program may be used on a system that does not include these routines in its library. However, it may be necessary to add a dummy subroutine with the entry-point names PLOT, SYMBOL, NUMBER, AXIS, and LINE, as some systems will not permit program execution with unsatisfied external references.

## 2. PROGRAM LOGIC

The complete program comprises of 42 FORTRAN programs and subprograms. They are distributed among the three principal overlays as follows:

a. Resident

Program UD0300. The main program, controls entry to each section.

b. Aerodynamic Section

Program UD03AR. The "main program" for the aerodynamic section, controls logic within the section.

Subroutine UD0301. Spline-fit or linear interpolation and gradient determination.

Subroutine UD0302. Reads input data.

| | |
|---|---|
| Subroutine UD0303. | Checks lines printed per page. |
| Subroutine UD0304. | Axisymmetric turbulent mixing calculations. |
| Subroutine UD0329. | Determines radial derivatives for mixing calculation. |
| Subroutine UD0305. | Handles interpolation of input data at a particular computing station. |
| Subroutine UD0306. | Modification of loss coefficients to account for radial blade-wake transfers. |
| Subroutine UD0330. | Solves simultaneous linear equations for least-square fit. |
| Subroutine UD0307. | Determines entropy, rothalpy or enthalpy, and whirl velocity at a computing station. |
| Subroutine UD0309. | Re-estimates loss coefficients. |
| Subroutine UD0310. | Annulus wall boundary layer calculation. |
| Subroutine UD0311. | Regular printed output. |
| Subroutine UD0312. | Precision-plot output. |
| Subroutine UDG1. | Reads data describing fluid. |
| Function UDG2. | Enthalpy = f (entropy, pressure) |
| Function UDG3. | Entropy = f (pressure, enthalpy) |
| Function UDG4. | Pressure = f (enthalpy, entropy) |
| Function UDG5. | Specific weight = f (enthalpy, entropy) |
| Function UDG6. | Enthalpy = f (pressure, temperature) |
| Function UDG7. | Temperature = f (enthalpy, entropy) |
| Function UDG8. | Ratio of specific heats = f (enthalpy, entropy) |
| Function UDG9. | Square of Mach number = f (static enthalpy, entropy, square of velocity) |

| | |
|---|---|
| Subroutine UD0325. | Creates line-printer plots. |
| Subroutine UD0308. | Determines simultaneous solution to momentum and continuity equations when NEQN = 0 or 1. |
| Subroutine UD0326 | Determines simultaneous solution to momentum and continuity equations when NEQN = 2 or 3. |

c. Analytic Meanline Blade Section

Program UD03AN. The "main program." for the analytic mean-line blade section. Corresponds to Program BLADE, Ref. 3.

| | |
|---|---|
| Subroutine UD0313. | Corresponds to Subroutine BQ, Ref. 3. |
| Subroutine UD0314. | Corresponds to Subroutine CQ, Ref. 3. |
| Subroutine UD0315. | Corresponds to Subroutine DI, Ref. 3. |
| Subroutine UD0316. | Corresponds to Subroutine EQ, Ref. 3. |
| Subroutine UD0317. | Corresponds to Subroutine FQ, Ref. 3. |
| Subroutine UD0318. | Corresponds to Subroutine GQ, Ref. 3. |
| Subroutine UD0319. | Determines deviation angles and formats data for aerodynamic section. |

d. Arbitrary Meanline Blade Section

Program UD03AB. The "main program" for the arbitrary mean-line blade section. Corresponds to Program ARBITR, Ref. 4.

| | |
|---|---|
| Subroutine UD0320. | Corresponds to Subroutine BQ, Ref. 4. |
| Subroutine UD0321. | Interfaces with Subroutine UD0327. |
| Subroutine UD0322. | Interfaces with Subroutine UD0327. |
| Subroutine UD0323. | Corresponds to Subroutine EQ, Ref. 4. |
| Subroutine UD0324. | Corresponds to Subroutine FQ, Ref. 4. |
| Subroutine UD0327. | Spline-fit routine for interpolation, gradients and integrals, also linear interpolation. |

The overall program logic is illustrated by the flow chart in Figure 2. This shows all the significant actions in Program UD0300, expressed by means of the FORTRAN variables and statement labels.

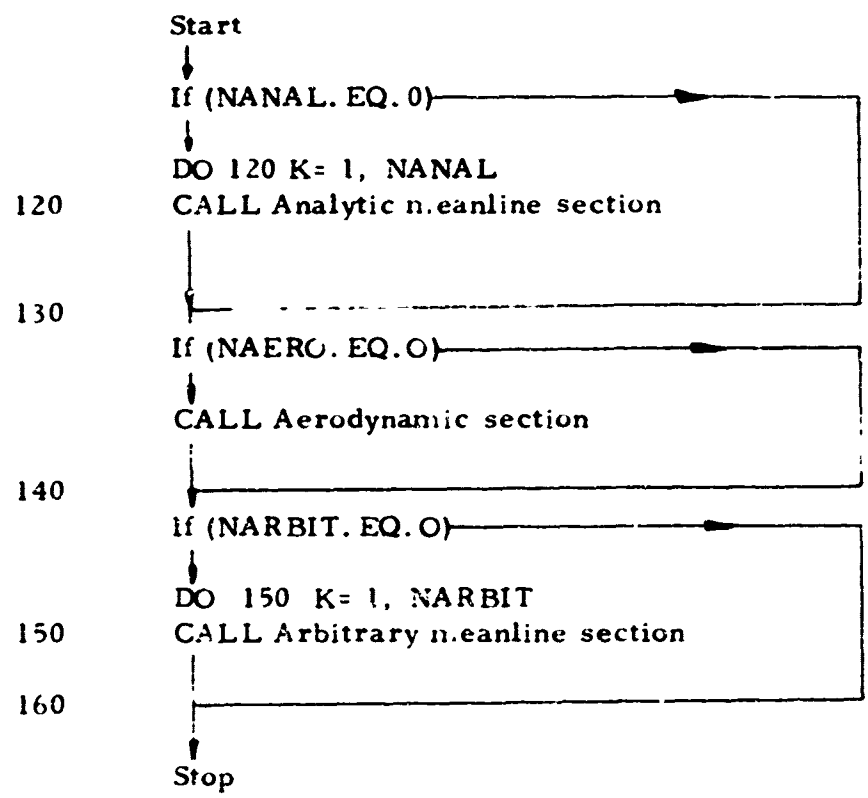


Figure 2. Overall Program Logic

Details of the operation of the analytic meanline section may be found in References 3 and 5. The only significant modification made in incorporating the program described therein into Program UD0300 is the addition of Subroutine UD0319. This subroutine is entered once as a final operation if data from the analytic meanline blade section are required as input for the aerodynamic section.

The arbitrary meanline blade section operates basically as described in References 4 and 5, but a new method of generating the meanline has been developed. Previously, a number of meanlines were examined, based upon a range of boundary conditions at the leading edge. The meanline was then selected from those obtained on the basis of the minimum number of inflection points and the largest minimum radius of

curvature on the meanline. The segment of meanline between each point where the blade angle was specified (a computing station in the aerodynamic analysis) was an algebraic cubic, so that the resulting meanline was a spline-curve restrained to give the desired blade angle at each computing station. At the leading edge, the radius of curvature of the meanline was defined. The new procedure is to fit a spline-curve through the specified angles at each computing station. Then the analytic integral of the spline curve gives the meanline which is therefore a piece-wise quartic. Boundary conditions used for the spline-fit are that the second derivative be zero at each end. The new procedure has been found to give meanlines with generally larger minimum radii of curvature than the old procedure, and, in some instances, to give a satisfactory meanline when none could be found using the old procedure.

The logic of the aerodynamic section is illustrated by the flow chart shown in Figure 3, which is essentially a flow chart for Program UD03AR. All significant actions and calls to subroutines (other than to Subroutine UD0301 for interpolation) in the program are shown.

3. PROGRAM AND SUBPROGRAM DESCRIPTIONS

a. Program UD0300. Cards identified by $MN$. This is the main program and controls entry to each section, as shown in Figure 2.

b. Program UD03AR. Cards identified by $AR$. This is the main routine of the aerodynamic section, and it controls the logic of the section, as shown in Figure 3. In terms of conventional FORTRAN it is a sub-routine, but is shown as a program because of the use of CDC Scope ove y. Some significant points are distributed in the program as follows.

| Operation | First Related Card |
|---|---|
| Call UD0302 to read input. | \$AR\$. 43 |
| Determine if streamline estimate is required. | \$AR\$. 51 |
| Begin streamline estimation. | \$AR\$. 54 |
| Determine geometric constant in relaxation factor. | \$AR\$. 152 |
| Interpolate inlet conditions | \$AR\$. 169 |
| Estimate inlet station velocity. | \$AR\$. 191 |
| Set integers to begin a pass. | \$AR\$. 196 |

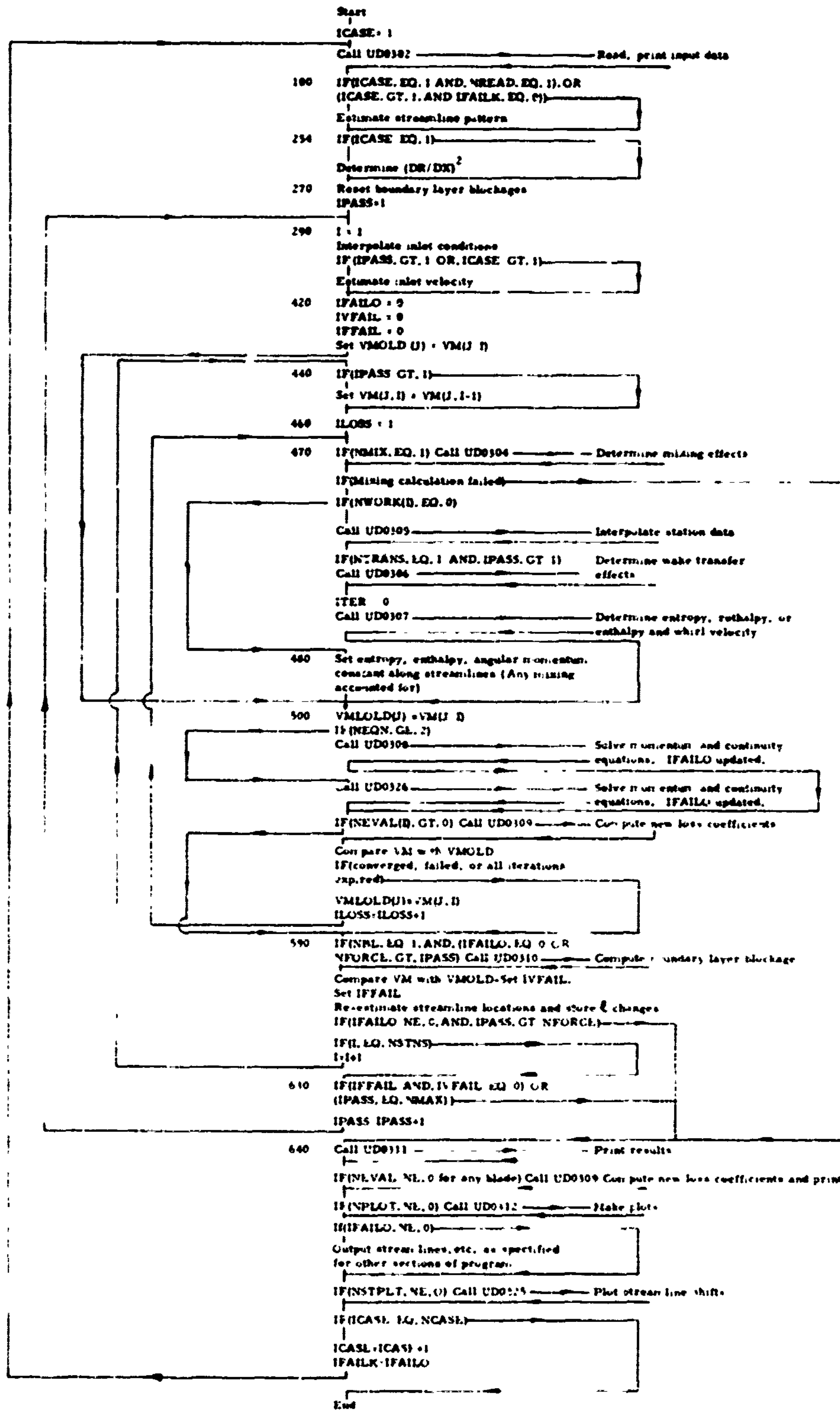

Figure 3. Logic of Aerodynamic Section.

| | |
|---|---|
| Optional call to UD0304 for turbulent mixing calculations. | $AR$. 213 |
| Call UD0305 to interpolate data at a bladed station. | $AR$. 215 |
| Optional call to UD0306 for blade wake transfer calculations. | $AR$. 216 |
| Call UD0307 to determine entropy, rothalpy, or enthalpy and whirl velocity. | $AR$. 218 |
| Set entropy, enthalpy, and angular momentum following blade free space. | $AR$. 220 |
| Call UD0308 to solve momentum and continuity equations, if NEQN = 0 or 1. | $AR$. 229 |
| Call UD0326 to solve momentum and continuity equations, if NEQN = 2 or 3. | $AR$. 231 |
| Optional call to UD0309 to re-estimate loss coefficients. | $AR$. 234 |
| Check convergence of meridional velocity in loss re-estimation loop. | $AR$. 236 |
| Optional call to UD0310 to compute annulus wall boundary layers. | $AR$. 258 |
| Check on overall convergence. | $AR$. 263 |
| Re-estimate streamline locations. | $AR$. 267 |
| Call UD0311 for printed output. | $AR$. 304 |
| Call UD0309 for printout of re-estimated loss coefficients if specified. | $AR$. 309 |
| Optional call to UD0312 for static pressure and streamline plots. | $AR$. 311 |
| Punch streamline pattern if specified. | $AR$. 314 |
| Punch strean line radii for analytic meanline section if specified. | $AR$. 318 |

| | |
|---|---|
| Output data for arbitrary meanline section if specified. | $AR$. 323 |
| Optional call to UD0325 to plot streamline relocation shifts. | $AR$. 348 |

c. Subroutine UD0301. Cards identified by $01$. Performs linear and splinefit interpolation and gradient determination.

d. Subroutine UD0302. Cards identified by $02$. Reads and prints (aerodynamic section) input data.

e. Subroutine UD0303. Cards identified by $03$. Checks and updates number of lines printed per page.

f. Subroutine UD0304. Cards identified by $04$. Turbulent mixing calculations.

g. Subroutine UD0329. Cards identified by $29$. Determines radial derivatives for mixing calculations.

h. Subroutine UD0305. Cards identified by $05$. This subroutine handles the interpolation of data prescribing conditions at a computing station. If relative flow angle is not specified, this is derived from the data.

i. Subroutine UD0306. Cards identified by $06$. Loss coefficients are modified to account for radial transfer of blade wakes.

j. Subroutine UD0330. Cards identified by $30$. Solves simultaneous linear equations.

k. Subroutine UD0307. Cards identified by $07$. If absolute angular momentum is specified (or implied) at a computing station, this subroutine determines total enthalpy, entropy, and whirl velocity. If relative flow angle is specified, this was determined in Subroutine UD0305, and here the rothalpy and entropy are determined. Some significant points in the program are distributed as follows.

| Operation | First Related Card |
|---|---|
| Determine enthalpy and entropy given total pressure and loss coefficient based on local dynamic head. | $07$. 33 |

| | |
|---|---|
| Determine enthalpy and entropy given total pressure and loss coefficient not based on local dynamic head. | $07$.46 |
| Determine enthalpy and entropy given total pressure and isentropic efficiency. | $07$.62 |
| Determine enthalpy and entropy given total pressure and entropy rise. | $07$.66 |
| Determine whirl velocity from enthalpy. | $07$.69 |
| Determine whirl velocity when enthalpy is given. | $07$.72 |
| Determine whirl velocity when angular momentum is given. | $07$.76 |
| Set whirl velocity when given directly. | $07$.79 |
| Determine enthalpy from whirl velocity. | $07$.81 |
| Determine entropy given loss coefficient based on local dynamic head and enthalpy, directly or derived. | $07$.85 |
| Determine entropy given loss coefficient not based on local dynamic head, and enthalpy, directly or derived. | $07$.97 |
| Determine entropy given isentropic efficiency and enthalpy, directly or derived. | $07$.112 |
| Determine entropy given entropy rise and enthalpy, directly or derived. | $07$.115 |
| Determine rothalpy, given relative flow angle. | $07$.118 |
| Determine entropy given loss coefficient based on local dynamic head and relative flow angle. | $07$.122 |
| Determine entropy given loss coefficient not based on local dynamic head and relative flow angle. | $07$.134 |

| | |
|---|---|
| Determine entropy given isentropic efficiency and relative flow angle. | \$07\$. 146 |
| Determine entropy given entropy rise and relative flow angle. | \$07\$. 158 |

1. Subroutine UD0309. Cards identified by \$09\$. This subroutine estimates the relative total pressure loss coefficients. If IPRINT = 0, the new values are placed in array DATA2; if IPRINT = 1, the new values are printed out. Some significant points in the subroutine are distributed as follows.

| Operation | First Related Card |
|---|---|
| A maximum value for the calculated loss coefficient of 0.7 is set. (This is subsequently only imposed when the value is incorporated into the calculation, not when it is printed out.) | \$09\$. 28 |
| Heading depending upon use of calculated loss coefficients is printed. | \$09\$. 35 |
| Cascade solidity is interpolated. | \$09\$. 61 |
| Diffusion factor is determined. | \$09\$. 77 |
| The loss parameters are interpolated from the 10%, 50%, and 90% of blade-height curves as a function of diffusion factor. | \$09\$. 82 |
| The loss parameters are interpolated as a function of radial location. | \$09\$. 93 |
| The diffusion component of the loss coefficient is determined. | \$09\$. 101 |
| The shock component of the loss coefficient is omitted if NDEL = 0. | \$09\$. 103 |
| The Prandtl-Meyer expansion angle in the input data is interpolated. | \$09\$. 105 |
| If the blade geometry at the leading edge is not given, the incidence angle is not determined. | \$09\$. 106 |

| | |
|---|---|
| The expansion angle is the input value plus the incidence angle (or zero). | $09$.132 |
| The inlet relative Mach number is determined. | $09$.139 |
| The Prandtl-Meyer angle following the expansion is determined. | $09$.142 |
| If the Prandtl-Meyer angle is less than zero, indicating compression to a Mach number of less than unity, the suction surface Mach number at shockwave impingement is set to unity. | $09$.144 |
| The Mach number at the shockwave impingement is determined iteratively. | $09$.149 |
| The mean Mach number at the shock wave is the average of the relative inlet Mach number (or unity, if the inlet value is less than unity) and the suction surface impingement-point Mach number. | $09$.163 |
| The mean Mach number is multiplied by the relative inlet Mach number if this was less than unity. | $09$.164 |
| No shock-loss is computed if the mean Mach number is less than unity. | $09$.165 |
| The shock-loss component of the loss coefficient is determined. | $09$.167 |
| The total loss coefficient is determined. | $09$.170 |
| If the calculated results are to be printed, they are not incorporated into the overall computation. | $09$.171 |
| The limiting value of loss coefficient is imposed, if appropriate. | $09$.188 |

m. Subroutine UD0310. Cards identified by $10$. This subroutine computes the blockage due to the annulus wall boundary layers.

n. Subroutine UD0311. Cards identified by $11$. This subroutine prints the regular output.

o. Subroutine UD0312. Cards identified by $12$. This subroutine makes plots of static pressure and the streamline pattern. All calls to CALCOMP software from the aerodynamic section occur in this subroutine.

p. Subroutine UDG1. Cards for UDG1 and the associated functions identified by $GN$. The constants describing the fluid are read in by Subroutine UDG1, and communicated to the eight functions for thermodynamic properties by means of Common storage area GAS. This subroutine and the eight associated functions may be modified to change the type of fluid from a perfect gas.

q. Functions UDG2 through UDG9. These eight functions give thermodynamic properties as listed in the previous subsection.

r. Subroutine UD0325. Cards identified by $25$. This subroutine makes line-printer plots of the change in $\ell$ -coordinates of the mid-streamline from pass to pass, for each station, if requested.

s. Subroutine UD0308. Cards identified by $08$. This is the most important subroutine in the aero section. It solves the momentum and continuity equations when NEQN = 0 or 1. Some significant points in the subroutine are distributed as follows.

| Operation | First Related Card |
|---|---|
| Determine streamline slope angle. | $08$.64 |
| Determine curvature of streamlines. | $08$.65 |
| Determine streamwise gradient of blade blockage. | $08$.68 |
| Determine streamwise gradient of entropy. | $08$.70 |
| Determine streamwise gradient of angular momentum. | $08$.71 |
| Determine streamline slope angle. | $08$.104<br>$08$.107 |
| Determine tangent of station lean angle. | $08$.131 |
| Determine gradient along computing station of entropy and streamline slope angle. | $08$.140 |

| | |
|---|---|
| Determine gradient along computing station of angular momentum and enthalpy. | \$08\$. 143 |
| Determine local sound speed for design case. | \$08\$. 148 |
| Determine gradient along computing station of $r \tan \beta$ and rothalpy. | \$08\$. 161 |
| Determine local sound speed for analysis case. | \$08\$. 165 |
| Select analysis case momentum equation solution. | \$08\$. 184 |
| Start design case momentum equation solution. Determine static temperature. | \$08\$. 185 |
| Determine parameter $A$ in momentum equation. | \$08\$. 220 |
| Determine parameter $B$ in momentum equation. | \$08\$. 224 |
| Integrate momentum equation. | \$08\$. 241 |
| Impose a maximum limit on any meridional velocity of 2.0 times the mid-streamline value. | \$08\$. 271 |
| Impose a minimum limit on any meridional velocity of 1.0. | \$08\$. 280 |
| New estimate of velocity is stored temporarily. | \$08\$. 292 |
| Meridional velocity is restrained to be not greater than sound speed. | \$08\$. 293 |
| Relaxation factor is determined and applied to give new velocity estimate. | \$08\$. 303 |
| Call UD0307 if entropy is a function of velocity. | \$08\$. 312 |
| Determine static enthalpy and square of meridional Mach number. | \$08\$. 313 |
| Start analytic case momentum equation solution. | \$08\$. 330 |
| Determine static temperature. | \$08\$. 342. |

| | |
|---|---|
| Determine meridional velocity gradient. | \$08\$. 381 |
| Determine meridional velocity on adjacent streamline. | \$08\$. 397 |
| Impose a maximum limit on the meridional velocity of 3.0 times the mid-streamline value. | \$08\$. 398 |
| Impose a minimum limit on the meridional velocity of 1.0. | \$08\$. 410 |
| Impose a maximum limit on the meridional velocity of the local sound speed. | \$08\$. 424 |
| Check convergence of meridional velocity. | \$08\$. 429 |
| If entropy is a function of velocity, call UD0307. | \$08\$. 452 |
| Determine static enthalpy and square of relative Mach number. | \$08\$. 454 |
| Start continuity equation solution. Integrate continuity equation and equation for gradient of flow with velocity. | \$08\$. 478 |
| Normalize fractions of flow in each streamline for streamline re-estimation. | \$08\$. 506 |
| Check for supersonic solution. | \$08\$. 508 |
| Determine meridional velocity increment. | \$08\$. 511 |
| Impose limits on velocity increment. | \$08\$. 512 |
| Add increment to meridional velocities, impose minimum value of 1.0, maximum value of sound speed. | \$08\$. 519 |
| Check convergence of momentum and continuity equations. | \$08\$. 555 |
| Call UD0307 if entropy is a function of velocity. | \$08\$. 565 |
| Determine relative flow angle for design cases. | \$08\$. 592 |

Determine enthalpy and whirl velocity for analysis cases. $08$.597

t. Subroutine UD0326. Cards identified by $26$. This subroutine is very similar to Subroutine UD0308, and it solves the momentum and continuity equations when NEQN = 1 or 2. It was created by modification of Subroutine UD0308, and so is structured identically. Subroutine UD0326 does not have the local sound speed as an upper limit on meridional velocity, and the A and B parameters of the momentum equation reflect the alternative formulations.

u. Program UD03AN. Cards identified by $AN$. This is the main routine of the analytic meanline blade section of the program. In terms of conventional FORTRAN it is a subroutine, but appears as a program here because of the use of CDC Scope overlay. See Program BLADE, Reference 3 for details.

v. Subroutine UD0313. Cards identified by $13$. See Subroutine BQ, Reference 3 for details.

w. Subroutine UD0314. Cards identified by $14$. See Subroutine CQ, Reference 3 for details.

x. Subroutine UD0315. Cards identified by $15$. See Subroutine D1, Reference 3 for details.

y. Subroutine UD0316. Cards identified by $16$. See Subroutine EQ, Reference 3 for details.

z. Subroutine UD0317. Cards identified by $17$. See Subroutine FQ, Reference 3 for details.

aa. Subroutine UD0318. Cards identified by $18$. See Subroutine GQ, Reference 3 for details.

ab. Subroutine UD0319. Cards identified by $19$. This subroutine determines deviation angles and hence relative flow angles, reads in other data regarding the blade, and puts onto a scratch file, (LOG5), data for the aerodynamic section.

ac. Program UD03AB. Cards identified by $AB$. This is the main routine of the arbitrary meanline blade section of the program. In terms of conventional FORTRAN it is a subroutine, but appears as a program here because of the use of CDC Scope overlay. See Program ARBITR, Reference 4 for details, except that some simplification occurs through use of the new meanline generation procedure.

ad. Subroutine UD0320. Cards identified by $20$. See Subroutine BQ, Reference 4 for details.

ae. Subroutine UD0321. Cards identified by $21$. Calls Subroutine UD0327.

af. Subroutine UD0322. Cards identified by $22$. Calls Subroutine UD0327.

ag. Subroutine UD0323. Cards identified by $23$. See Subroutine EQ, Reference 4 for details.

ah. Subroutine UD0324. Cards identified by $24$. See Subroutine FQ, Reference 4 for details.

ai. Subroutine UD0327. Cards identified by $27$. This subroutine provides interpolation, derivatives and integrals based upon spline fitting, and also linear interpolation.

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## LIST OF SYMBOLS

| | |
|---|---|
| $A$ | Parameter in momentum equation |
| $a/c$ | Ratio of distance to maximum camber point to blade chord |
| $B$ | Parameter in momentum equation |
| $B$ | Blockage due to annulus wall boundary layers and blade wakes |
| $b$ | Exponent in deviation rule, from Figure 164, Reference 6 |
| $c$ | Blade chord |
| $D$ | Diffusion factor |
| $E$ | Arbitrary error in streamline location |
| $F$ | Optimum streamline relocation relaxation factor |
| $F_r$ | Radial force on element of boundary layer |
| $F$ | Radial movement of boundary layer material |
| $g$ | Acceleration due to gravity |
| $H$ | Total enthalpy |
| $H$ | Boundary layer shape factor, ratio of displacement to momentum thicknesses |
| $h$ | Static enthalpy |
| $I$ | Rothalpy, defined by Equation 48 |
| $J$ | Joules equivalent |
| $K_{sh}$ | Shape factor in deviation rule |
| $K_{tc}$ | Thickness factor in deviation rule, from Figure 172, Reference 6 |
| $\ell$ | Distance along computing station outwards from hub |
| $M$ | Mach number |
| $M_m$ | Ratio of meridional velocity component to speed of sound |
| $M_\theta$ | Ratio of tangential velocity component to speed of sound |
| $M_r$ | Ratio of relative velocity to speed of sound |
| $m$ | Meridional distance in direction of flow |
| $P$ | Total pressure |
| $P'$ | Ideal (isentropic) total pressure |
| $P_r$ | Relative total pressure |

## LIST OF SYMBOLS (Continued)

| | |
|---|---|
| $p$ | Static pressure |
| $Q$ | Rate of change of flow with meridional velocity at mid-radius |
| $R$ | Streamline relocation relaxation factor |
| $r$ | Radius |
| $r_c$ | Radius of curvature of streamsurface projected onto meridional plane |
| $S$ | Entropy |
| $T$ | Total temperature |
| $t$ | Static temperature |
| $U_\infty$ | Free-stream velocity |
| $V$ | Velocity |
| $V_r$ | Radial component of velocity |
| $V_z$ | Axial component of velocity |
| $V_m$ | Meridional component of velocity |
| $V_\theta$ | Tangential or whirl component of velocity, positive in direction of rotor rotation |
| $W$ | Flow rate |
| $W$ | Relative velocity |
| $W_\theta$ | Relative tangential velocity component |
| $\omega$ | Specific weight |
| $\omega_T$ | Specific weight based on total conditions |
| $x$ | Axial coordinate |
| $\alpha$ | Absolute whirl angle, defined by Equation 1 |
| $\beta$ | Relative flow angle, defined by $\tan\beta = W_\theta / V_m$ |
| $\gamma$ | Ratio of specific heats |
| $\gamma$ | Station lean angle, defined by $\tan\gamma = Dx/D\ell$ |
| $\delta$ | Deviation angle |
| $\delta_{o_{10}}$ | Reference deviation from Figure 161, Reference 6 |
| $\epsilon$ | Blade lean angle on a computing station, defined by $\frac{\tan\epsilon}{r} = -\frac{D\theta}{Dx}$ with $\theta$ increasing in the direction of rotor rotation |

## LIST OF SYMBOLS (Continued)

| | |
|---|---|
| $\xi$ | Blade lean angle projected onto constant-x plane, defined by Equation 61 |
| $\xi$ | Eddy viscosity |
| $\eta$ | Isentropic efficiency |
| $\theta$ | Camber angl |
| $\Theta$ | Circumferential coordinate |
| $\theta$ | Boundary layer momentum thickness |
| $\lambda$ | Area coefficient, defined (1- Blade blockage) |
| $\nu$ | Kinematic viscosity |
| $\rho$ | Density |
| $\sigma$ | Cascade solidity |
| $\tau$ | Boundary layer shear stress |
| $\phi$ | Streamline slope angle, defined by $\tan\phi = \frac{Dr}{Dx}$ |
| $\omega$ | Speed of rotation of blading and coordinate system |
| $\bar{\omega}$ | Relative total pressure loss coefficient |



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